Vivekanand Education Society's

College of Arts, Science and Commerce

(Autonomous)

Sindhi Society, Chembur, Mumbai, Maharashtra– 400 071.

Accredited by NAAC"A Grade"in3rd Cycle - 2017
Best College Award – Urban Area, University of Mumbai (2012-13)
Recipient of FIST Grant (DST) and STAR College Grant (DBT)

Affiliated to the

University of Mumbai

Syllabus

For S.Y.B.Sc. Semester III & IV

Program: B.Sc. (Mathematics)

(Program code: VESUSMT)

As per Choice Based Semester and Grading System (CBSGS)
with effect from Academic Year 2023 - 2024

Program Outcomes (PO):

A leaner completing B.Sc.(Mathematics) will be able to:

PO1 Demonstrate analytical skills in applying appropriate science principles and methodologies to solve a wide range of problems.

PO2 Design, carry out experiments and analyze results by accounting uncertainties in different quantities measured using various scientific instruments.

PO3 Demonstrate professional behavior of being unbiased, and truthful in all aspects of work as an individual as well as team.

PO4 Ability to communicate science effectively by written, computational and graphic means .

Program Specific Outcomes (PSO's)

On completion of B.Sc. (Mathematics) program, learners will be enriched with knowledge and be able to

PSO1 Think in a critical manner.

PSO2 Know when there is a need for information, to be able to identify, locate, evaluate, and effectively use that information for the issue or problem at hand.

PSO3 Formulate and develop mathematical arguments in a logical manner.

PSO4 Acquire good knowledge and understanding in advanced areas of mathematics chosen by the student from the given courses.

PSO5 Understand, formulate and use quantitative models arising in social science, business and other contexts.

S.Y.B.Sc. (Mathematics)

SEMESTER III

CALCULUS III				
Course Code	UNIT	TOPICS	Credits	L / Week
VESUSMT301	I	Infinite Series	2	3
	II	Riemann Integration		
	III	Applications of Integrations and Improper Integrals		
LINEAR ALGEBRA I				
VESUSMT302	I	System of Equations and Matrices	2	3
	II	Vector Spaces over R		
	III	Determinants, Linear Equations (Revisited)		
ORDINARY DIFFERENTIAL EQUATIONS				
VESUSMT303	I	Higher Order Linear Differential Equations	*2*	*3*
	II	Systems of First Order Linear Differential Equations		
	III	Numerical Solutions of Ordinary Differential Equations		
PRACTICALS				
VESUSMTP03	Practicals based on VESUSMT301, VESUSMT302 and VESUSMT303		3	5

SEMESTER IV

MULTIVARIABLE CALCULUS I				
Course Code	UNIT	TOPICS	Credits	L / Week
VESUSMT401	I	Functions of Several Variables	2	3
	II	Differentiation of Scalar Fields		
	III	Applications of Differentiation of Scalar Fields and Differentiation of Vector Fields		
LINEAR ALGEBRA II				
VESUSMT402	I	Linear Transformation, Isomorphism, Matrix Associated With L. T.	2	3
	II	Inner Product Spaces		
	III	Eigen Values, Eigen Vectors, Diagonalizable Matrix		
NUMERICAL METHODS				
VESUSMT403	I	Solutions of Algebraic and Transcendental Equations	*2*	*3*
	II	Interpolation, Curve Fitting, Numerical Integration		
	III	Solutions of Linear System of Equations and Eigen Value Problems		
PRACTICALS				
VESUSMTP04	Practicals based on VESUSMT401, VESUSMT402 and VESUSMT403		3	5

Objectives:

(1) Give the students a sufficient knowledge of fundamental principles, methods and a clear perception of innumerous power of mathematical ideas and tools and know low to use the by modeling, solving and interpreting.
(2) reflecting the broad nature of the subset and developing mathematical tools for continuing further study in various fields of science.
(3) Enhancing students' overall development and to equip them with mathematical modeling abilities, problem solving skills, creative talent and power of communication necessary for various kinds of employment.
(4) A student should get adequate exposure to global and local concerns that explore them many aspects of Mathematical Sciences.

Learning Outcomes:

Calculus III:

On successful completion of this course, students will be able to:

1. Identify the different types of Infinite Series and Determine if an infinite series is convergent or divergent by selecting the appropriate tests such as Comparison test, Limit form of comparison test, Leibnitz's test for alternating series, D' Alemberts ratio test, Cauchy nth root test etc.
2. Define Riemann integration via approximation of areas and will be able to identify integrable and non-integrable functions.
3. Find area between the curves, arch length, surface area and solve problems of Type I and Type II integrals using comparison test, Beta and Gamma functions and their properties.

Multivariable Calculus I:

On successful completion of this course, students will be able to:

1. Examine real and vector valued functions of several variables. Define and compute Limits and continuity of real and vector valued functions. Define and compute partial and directional derivatives of scalar valued functions.
2. Define differentiability of scalar fields, calculate total derivatives of functions of two and three variables and learn basic properties of differentiability.
3. Find local extreme values of functions of several variables, test for saddle points, solve constraint problems using Lagrange multipliers methods and solve related application problems. Define Differentiability of vector fields and relationship between total derivative and Jacobian matrix.

Linear Algebra (I & II):

On successful completion of this course, students will be able to:

1. Solve systems of linear equations using multiple methods, including Gaussian elimination and matrix inversion.

2. Carry out matrix operations, including inverses and determinants.
3. Demonstrate understanding of the concepts of vector space and subspace.
4. Demonstrate understanding of linear independence, span, and basis.
5. Determine eigenvalues and eigenvectors and solve eigenvalue problems.
6. Apply principles of matrix algebra to linear transformations.
7. Demonstrate understanding of inner products and associated norms.

Ordinary Differential Equations:

On successful completion of this course:

1. Students will be able to find the complete solution of a Homogeneous and nonhomogeneous differential equation as a linear combination of the complementary function and a particular solution. Students will learn to solve linear differential equations with constant coefficients by the method of undetermined coefficients and method of constant coefficients by variation of parameters.
2. Students will learn to solve systems of homogeneous and nonhomogeneous linear differential equations.
3. Students will be able to find numerical solutions of initial value problems of first order and higher order ordinary differential equations using Taylor's expansion, Euler's Method,Picard's Method, Runge-Kutta method. Students will be able to solve simultaneous and higher order differential equations using the RK method and finite difference method.

Numerical Methods:

On successful completion of this course:

1. Students will be able to solve an algebraic or transcendental equation, Approximate a function and solve a differential equation using an approximate numerical method.
2. Students will learn to evaluate a derivative at a value, Solve a linear system of equations using an appropriate numerical method, Perform an error analysis for a given numerical method and Prove results for numerical root finding methods.
3. Students will be able to calculate a definite integral using an appropriate numerical method and Code a numerical method in a modern computer language.

Teaching Pattern for Semester III

(1) Three lectures per week per course.
(2) One Practical (2L) per week per batch for course VESUSMT301, VESUSMT302 combined and one practical (3L) per week per batch for course VESUSMT303.

Teaching Pattern for Semester IV

(1) Three lectures per week per course.
(3) One Practical (2L) per week per batch for course VESUSMT401, VESUSMT402 combined and one practical (3L) per week per batch for course VESUSMT403.

Semester III

Note: Unless indicated otherwise, proofs of the results mentioned in the syllabus should be covered.

VESUSMT301:Calculus III

Unit I. Infinite Series (15 Lectures)

1. Infinite series in $\mathbb{R}$. Definition of convergence and divergence. Basic examples including geometric series. Elementary results such as if $\sum_{n=1}^{\infty} a_n$ is convergent, then $a_n \longrightarrow 0$ but converse not true. Cauchy Criterion. Algebra of convergent series.

2. Tests for convergence: Comparison Test, Limit Comparison Test, Ratio Test (without proof), Root Test (without proof), Abel Test (without proof) and Dirichlet Test (without proof). Examples. The decimal expansion of real numbers. Convergence of $\sum_{n=1}^{\infty} \frac{1}{n^p}$ $(p > 1)$. Divergence of harmonic series $\sum_{n=1}^{\infty} \frac{1}{n}$.

3. Alternating series. Leibnitz's Test. Examples. Absolute convergence, absolute convergence implies convergence but not conversely. Conditional Convergence.

Unit II. Riemann Integration (15 Lectures)

1. Idea of approximating the area under a curve by inscribed and circumscribed rectangles. Partitions of an interval. Refinement of a partition. Upper and Lower sums for a bounded real valued function on a closed and bounded interval. Riemann integrability and the Riemann integral.

2. Criterion for Riemann integrability. Characterization of the Riemann integral as the limit of a sum. Examples.

3. Algebra of Riemann integrable functions. Also, basic results such as if $f : [a,b] \longrightarrow \mathbb{R}$ is integrable, then (i) $\int_a^b f(x)\ dx = \int_a^c f(x)\ dx + \int_c^b f(x)\ dx$. (ii) $|f|$ is integrable and $\left| \int_a^b f(x)\ dx \right| \leq \int_a^b |f|(x)\ dx$ (iii) If $f(x) \geq 0$ for all $x \in [a,b]$ then $\int_a^b f(x)\ dx \geq 0$.

4. Riemann integrability of a continuous function, and more generally of a bounded function whose set of discontinuities has only finitely many points. Riemann integrability of monotone functions.

Unit III. Applications of Integrations and Improper Integrals (15 lectures)

1. Area between the two curves. Lengths of plane curves. Surface area of surfaces of revolution.
2. Continuity of the function $F(x) = \int_a^x f(t)\ dt, x \in [a, b]$, when $f : [a, b] \longrightarrow \mathbb{R}$ is Riemann integrable. First and Second Fundamental Theorems of Calculus.
3. Mean value theorem. Integration by parts formula. Leibnitz's Rule.
4. Definition of two types of improper integrals. Necessary and sufficient conditions for convergence.
5. Absolute convergence. Comparison and limit comparison tests for convergence.
6. Gamma and Beta functions and their properties. Relationship between them (without proof).

Reference Books

1. Sudhir Ghorpade, Balmohan Limaye; A Course in Calculus and Real Analysis (second edition); Springer.
2. R.R. Goldberg; Methods of Real Analysis; Oxford and IBH Pub. Co., New Delhi, 1970.
3. Calculus and Analytic Geometry (Ninth Edition); Thomas and Finney; Addison-Wesley, Reading Mass., 1998.
4. T. Apostol; Calculus Vol. 2; John Wiley.

Additional Reference Books

1. Ajit Kumar, S.Kumaresan; A Basic Course in Real Analysis; CRC Press, 2014
2. D. Somasundaram and B.Choudhary; A First Course in Mathematical Analysis, Narosa, New Delhi, 1996.
3. K. Stewart; Calculus, Booke/Cole Publishing Co, 1994.
4. J. E. Marsden, A.J. Tromba and A. Weinstein; Basic Multivariable Calculus; Springer.
5. R.G. Brtle and D. R. Sherbert; Introduction to Real Analysis Second Ed. ; John Wiley, New Yorm, 1992.
6. M. H. Protter; Basic Elements of Real Analysis; Springer-Verlag, New York, 1998.

VESUSMT302: LINEAR ALGEBRA I

Unit I. System of Equations, Matrices (15 Lectures)

1. Systems of homogeneous and non-homogeneous linear equations, Simple examples of finding solutions of such systems. Geometric and algebraic understanding of the solutions. Matrices (with real entries), Matrix representation of system of homogeneous and non-homogeneous linear equations. Algebra of solutions of systems of homogeneous linear equations. A system of homogeneous linear equations with number of unknowns more than the number of equations has infinitely many solutions.

2. Elementary row and column operations. Row equivalent matrices. Row reduction (of a matrix to its row echelon form). Gaussian elimination. Applications to solving systems of linear equations. Examples.

3. Elementary matrices. Relation of elementary row operations with elementary matrices. Invertibility of elementary matrices. Consequences such as (i) a square matrix is invertible if and only if its row echelon form is invertible. (ii) invertible matrices are products of elementary matrices. Examples of the computation of the inverse of a matrix using Gauss elimination method.

Unit II. Vector space over $\mathbb{R}$ (15 Lectures)

1. Definition of a vector space over $\mathbb{R}$. Subspaces; criterion for a nonempty subset to be a subspace of a vector space. Examples of vector spaces, including the Euclidean space $\mathbb{R}^n$, lines, planes and hyperplanes in $\mathbb{R}^n$ passing through the origin, space of systems of homogeneous linear equations, space of polynomials, space of various types of matrices, space of real valued functions on a set.

2. Intersections and sums of subspaces. Direct sums of vector spaces. Quotient space of a vector space by its subspace.

3. Linear combination of vectors. Linear span of a subset of a vector space. Definition of a finitely generated vector space. Linear dependence and independence of subsets of a vector space.

4. Basis of a vector space. Basic results that any two bases of a finitely generated vector space have the same number of elements. Dimension of a vector space. Examples. Bases of a vector space as a maximal linearly independent sets and as minimal generating sets.

Unit III. Determinants, Linear Equations (Revisited) (15 Lectures)

1. Inductive definition of the determinant of a $n \times n$ matrix (e. g. in terms of expansion along the first row). Example of a lower triangular matrix. Laplace expansions along an arbitrary row or column. Determinant expansions using permutations $\left(\det(A) = \sum_{\sigma \in S_n} \operatorname{sign}(\sigma) \prod_{i=1}^{n} a_{\sigma(i),i}\right)$.

2. Basic properties of determinants (Statements only); (i) $\det A = \det A^T$. (ii) Multilinearity and alternating property for columns and rows. (iii) A square matrix A is invertible if and only if $\det A \neq 0$. (iv) Minors and cofactors. Formula for A^{-1} when $\det A \neq 0$. (v) $\det(AB) = \det A \det B$.

3. Row space and the column space of a matrix as examples of vector space. Notion of row rank and the column rank. Equivalence of the row rank and the column rank. Invariance of rank upon elementary row or column operations. Examples of computing the rank using row reduction.

4. Relation between the solutions of a system of non-homogeneous linear equations and the associated system of homogeneous linear equations. Necessary and sufficient condition for a system of non-homogeneous linear equations to have a solution [viz., the rank of the coefficient matrix equals the rank of the augmented matrix $[A|B]$]. Equivalence of statements (in which A denotes an $n \times n$ matrix) such as the following.

   (i) The system $A\boldsymbol{x} = \boldsymbol{b}$ of non-homogeneous linear equations has a unique solution.

   (ii) The system $A\boldsymbol{x} = \mathbf{0}$ of homogeneous linear equations has no nontrivial solution.

   (iii) A is invertible.

   (iv) $\det A \neq 0$.

   (v) $\text{rank}(A) = n$.

5. Cramers Rule. LU Decomposition. If a square matrix A is a matrix that can be reduced to row echelon form U by Gauss elimination without row interchanges, then A can be factored as $A = LU$ where L is a lower triangular matrix.

Reference books

1 Howard Anton, Chris Rorres, Elementary Linear Algebra, Wiley Student Edition).

2 Serge Lang, Introduction to Linear Algebra, Springer.

3 S Kumaresan, Linear Algebra - A Geometric Approach, PHI Learning.

4 Sheldon Axler, Linear Algebra done right, Springer.

5 Gareth Williams, Linear Algebra with Applications, Jones and Bartlett Publishers.

6 David W. Lewis, Matrix theory.

VESUSMT303: ORDINARY DIFFERENTIAL EQUATIONS

Unit I. Higher order Linear Differential equations (15 Lectures)

1. The general $n-$th order linear differential equations, Linear independence, An existence and uniqueness theorem, the Wronskian, Classification: homogeneous and non-homogeneous, General solution of homogeneous and non-homogeneous LDE, The Differential operator and its properties.

2. Higher order homogeneous linear differential equations with constant coefficients, the auxiliary equations, Roots of the auxiliary equations: real and distinct, real and repeated, complex and complex repeated.

3. Higher order homogeneous linear differential equations with constant coefficients, the method of undermined coefficients, method of variation of parameters.

4. The inverse differential operator and particular integral, Evaluation of $\frac{1}{f(D)}$ for the functions like e^{ax}, $\sin ax$, $\cos ax$, x^m, $x^m \sin ax$, $x^m \cos ax$, $e^{ax}V$ and xV where V is any function of x,

5. Higher order linear differential equations with variable coefficients:
   The Cauchy's equation: $x^3\frac{d^3y}{dx^3} + x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = f(x)$ and
   The Legendre's equation: $(ax+b)^3\frac{d^3y}{dx^3} + (ax+b)^2\frac{d^2y}{dx^2} + (ax+b)\frac{dy}{dx} + y = f(x)$.

Reference Books

1. Units 5, 6, 7 and 8 of E.D. Rainville and P.E. Bedient; Elementary Differential Equations; Macmillan.

2. Units 5, 6 and 7 of M.D. Raisinghania; Ordinary and Partial Differential Equations; S. Chand.

Unit II. Systems of First Order Linear Differential Equations (15 Lectures)

(a) Existence and uniqueness theorem for the solutions of initial value problems for a system of two first order linear differential equations in two unknown functions x, y of a single independent variable t, of the form $\begin{cases} \frac{dx}{dt} = F(t,x,y) \\ \frac{dy}{dt} = G(t,x,y) \end{cases}$ (Statement only).

(b) Homogeneous linear system of two first order differential equations in two unknown functions of a single independent variable t, of the form $\begin{cases} \frac{dx}{dt} = a_1(t)x + b_1(t)y, \\ \frac{dy}{dt} = a_2(t)x + b_2(t)y. \end{cases}$.

(c) Wronskian for a homogeneous linear system of first order linear differential equations in two functions x, y of a single independent variable t. Vanishing properties of the Wronskian. Relation with linear independence of solutions.

(d) Homogeneous linear systems with constant coefficients in two unknown functions x, y of a single independent variable t. Auxiliary equation associated to a homogenous system of equations with constant coefficients. Description fo the general solution depending on the roots and their multiplicities of the auxiliary equation, proof of independence of the solutions. Real form of solutions in case the auxiliary equation has complex roots.

(e) Non-homogeneous linear system of linear system of two first order differential equations in two unknown functions of a single independent variable t, of the form
$$\begin{cases} \frac{dx}{dt} = a_1(t)x + b_1(t)y + f_1(t), \\ \frac{dy}{dt} = a_2(t)x + b_2(t)y + f_2(t). \end{cases}$$
General Solution of non-homogeneous system. Relation between the solutions of a system

of non-homogeneous linear differential equations and the associated system of homogeneous linear differential equations.

Reference Books

1. G.F. Simmons; Differential Equations with Applications and Historical Notes; Taylor's and Francis.

Unit III. Numerical Solution of Ordinary Differential Equations (15 lectures)

1. Numerical Solution of initial value problem of first order ordinary differential equation using:
   (i) Taylor's series method,
   (ii) Picard's method for successive approximation and its convergence,
   (iii) Euler's method and error estimates for Euler's method,
   (iv) Modified Euler's Method,
   (v) Runge-Kutta method of second order and its error estimates,
   (vi) Runge-Kutta fourth order method.
2. Numerical solution of simultaneous and higher order ordinary differential equation using:
   (i) Runge-Kutta fourth order method for solving simultaneous ordinary differential equation,
   (ii) Finite difference method for the solution of two point linear boundary value problem.

Reference Books

1. Units 8 of S. S. Sastry, Introductory Methods of Numerical Analysis, PHI.

Additional Reference Books

1. E.D. Rainville and P.E. Bedient, Elementary Differential Equations, Macmillan.
2. M.D. Raisinghania, Ordinary and Partial Differential Equations, S. Chand.
3. G.F. Simmons, Differential Equations with Applications and Historical Notes, Taylor's and Francis.
4. S. S. Sastry, Introductory Methods of Numerical Analysis, PHI.
5. K. Atkinson, W.Han and D Stewart, Numerical Solution of Ordinary Differential Equations, Wiley.

VESUSMTP03: Practicals

Suggested Practicals for VESUSMTP301

1. Examples of convergent / divergent series and algebra of convergent series.
2. Tests for convergence of series.
3. Calculation of upper sum, lower sum and Riemann integral.
4. Problems on properties of Riemann integral.
5. Problems on fundamental theorem of calculus, mean value theorems, integration by parts, Leibnitz rule.
6. Convergence of improper integrals, different tests for convergence. Beta Gamma Functions.
7. Miscellaneous Theoretical Questions based on full paper.

Suggested Practicals for VESUSMTP302

1. Systems of homogeneous and non-homogeneous linear equations.
2. Elementary row/column operations and Elementary matrices.
3. Vector spaces, Subspaces.
4. Linear Dependence/independence, Basis, Dimension.
5. Determinant and Rank of a matrix.
6. Solution to a system of linear equations, LU decomposition
7. Miscellaneous Theory Questions.
8. Miscellaneous theory questions from units I, II and III.

Suggested Practicals for VESUSMTP303

1. Finding the general solution of homogeneous and non-homogeneous higher order linear differential equations.
2. Solving higher order linear differential equations using method of undetermined coefficients and method of variation of parameters.
3. Solving a system of first order linear ODES have auxiliary equations with real and complex roots.
4. Finding the numerical solution of initial value problems using Taylor's series method, Picard's method, modified Euler's method, Runge-Kutta method of fourth order and calculating their accuracy.
5. Finding the numerical solution of simultaneous ordinary differential equation using fourth order Runge-Kutta method.
6. Finding the numerical solution of two point linear boundary value problem using Finite difference method.

Semester IV

Note: Unless indicated otherwise, proofs of the results mentioned in the syllabus should be covered.

VESUSMT401 : MULTIVARIABLE CALCULUS I

UNIT I. Functions of Several Variables (15 Lectures)

1. Review of vectors in $\mathbb{R}^n$ [with emphasis on $\mathbb{R}^2$ and $\mathbb{R}^3$] and basic notions such as addition and scalar multiplication, inner product, length (norm), and distance between two points.
2. Real-valued functions of several variables (Scalar fields). Graph of a function. Level sets (level curves, level surfaces, etc). Examples. Vector valued functions of several variables (Vector fields). Component functions. Examples.
3. Sequences, Limits and Continuity: Sequence in $\mathbb{R}^n$ [with emphasis on $\mathbb{R}^2$ and $\mathbb{R}^3$] and their limits. Neighbourhoods in $\mathbb{R}^n$. Limits and continuity of scalar fields. Composition of continuous functions. Sequential characterizations. Algebra of limits and continuity (Results with proofs). Iterated limits.
   Limits and continuity of vector fields. Algebra of limits and continuity vector fields. (without proofs).
4. Partial and Directional Derivatives of scalar fields: Definitions of partial derivative and directional derivative of scalar fields (with emphasis on $\mathbb{R}^2$ and $\mathbb{R}^3$). Mean Value Theorem of scalar fields.

UNIT II. Differentiation of Scalar Fields (15 Lectures)

1. Differentiability of scalar fields (in terms of linear transformation). The concept of (total) derivative. Uniqueness of total derivative of a differentiable function at a point. Examples of functions of two or three variables. Increment Theorem. Basic properties including (i) continuity at a point of differentiability, (ii)existence of partial derivatives at a point of differentiability, and (iii) differentiability when the partial derivatives exist and are continuous.
2. Gradient. Relation between total derivative and gradient of a function. Chain rule. Geometric properties of gradient. Tangent planes.
3. Euler's Theorem.
4. Higher order partial derivatives. Mixed Partial Theorem (n=2).

UNIT III. Applications of Differentiation of Scalar Fields and Differentiation of Vector Fields (15 lectures)

1. Applications of Differentiation of Scalar Fields: The maximum and minimum rate of change of scalar fields. Taylor's Theorem for twice continuously differentiable functions. Notions of local maxima, local minima and saddle points. First Derivative Test. Examples. Hessian matrix. Second Derivative Test for functions of two variables. Examples. Method of Lagrange Multipliers.

2. Differentiation of Vector Fields: Differentiability and the notion of (total) derivative. Differentiability of a vector field implies continuity, Jacobian matrix. Relationship between total derivative and Jacobian matrix. The chain rule for derivative of vector fields (statements only).

Reference books

1. T. Apostol; Calculus, Vol. 2 (Second Edition); John Wiley.

2. Sudhir Ghorpade, Balmohan Limaye; A Course in Multivariable Calculus and Analysis (Second Edition); Springer.

3. Walter Rudin; Principles of Mathematical Analysis; McGraw-Hill, Inc.

4. J. E. Marsden, A.J. Tromba and A. Weinstein, Basic Multivariable Calculus; Springer.

5. D.Somasundaram and B.Choudhary; A First Course in Mathematical Analysis, Narosa, New Delhi, 1996.

6. K. Stewart; Calculus; Booke/Cole Publishing Co, 1994.

Additional Reference Books

1. Calculus and Analytic Geometry, G.B. Thomas and R. L. Finney, (Ninth Edition); Addison-Wesley, 1998.

2. Howard Anton; Calculus- A new Horizon,(Sixth Edition); John Wiley and Sons Inc, 1999.

3. S L Gupta and Nisha Rani; Principles of Real Analysis; Vikas Publishing house PVT LTD.

4. Shabanov, Sergei; Concepts in Calculus, III: Multivariable Calculus; University Press of Florida, 2012.

5. S C Malik and Savita Arora; Mathematical Analysis; New Age International Publishers.

VESUSMT402 : LINEAR ALGEBRA II

UNIT I. Linear Transformations

1. Definition of a linear transformation of vector spaces; elementary properties. Examples. Sums and scalar multiples of linear transformations. Composites of linear transformations. A Linear transformation of $V \longrightarrow W$, where V, W are vector spaces over $\mathbb{R}$ and V is a finite-dimensional vector space is completely determined by its action on an ordered basis of V.

2. Null-space (kernel) and the image (range) of a linear transformation. Nullity and rank of a linear transformation. Rank-Nullity Theorem (Fundamental Theorem of Homomorphisms).

3. Matrix associated with linear transformation of $V \longrightarrow W$ where V and W are finite dimensional vector spaces over $\mathbb{R}$.. Matrix of the composite of two linear transformations. Invertible linear transformations (isomorphisms), Linear operator, Effect of change of bases on matrices of linear operator.

4. Equivalence of the rank of a matrix and the rank of the associated linear transformation. Similar matrices.

UNIT II. Inner Products and Orthogonality

1. Inner product spaces (over $\mathbb{R}$). Examples, including the Euclidean space $\mathbb{R}^n$ and the space of real valued continuous functions on a closed and bounded interval. Norm associated to an inner product. Cauchy-Schwarz inequality. Triangle inequality.

2. Angle between two vectors. Orthogonality of vectors. Pythagoras theorem and some geometric applications in $\mathbb{R}^2$. Orthogonal sets, Orthonormal sets. Gram-Schmidt orthogonalizaton process. Orthogonal basis and orthonormal basis for a finite-dimensional inner product space.

3. Orthogonal complement of any set of vectors in an inner product space. Orthogonal complement of a set is a vector subspace of the inner product space. Orthogonal decomposition of an inner product space with respect to its subspace. Orthogonal projection of a vector onto a line (one dimensional subspace). Orthogonal projection of an inner product space onto its subspace.

UNIT III. Eigenvalues, Eigenvectors and Diagonalisation

1. Eigenvalues and eigenvectors of a linear transformation of a vector space into itself and of square matrices. The eigenvectors corresponding to distinct eigenvalues of a linear transformation are linearly independent. Eigen spaces. Algebraic and geometric multiplicity of an eigenvalue.

2. Characteristic polynomial. Properties of characteristic polynomials (only statements). Examples. Cayley-Hamilton Theorem. Applications.

3. Invariance of the characteristic polynomial and eigenvalues of similar matrices.

4. Diagonalisable matrix. A real square matrix A is diagonalisable if and only if there is a basis of $\mathbb{R}^n$ consisting of eigenvectors of A. (Statement only - $A_{n\times n}$ is diagonalisable if and only if sum of algebraic multiplicities is equal to sum of geometric multiplicities of all the eigenvalues of $A \ = n$). Procedure for diagonalising a matrix.

5. Spectral Theorem for Real Symmetric Matrices (Statement only). Examples of orthogonal diagonalisation of real symmetric matrices. Applications to quadratic forms and classification of conic sections.

Reference books

1. Howard Anton, Chris Rorres; Elementary Linear Algebra; Wiley Student Edition).

2. Serge Lang; Introduction to Linear Algebra; Springer.

3. S Kumaresan; Linear Algebra - A Geometric Approach; PHI Learning.

4. Sheldon Axler; Linear Algebra done right; Springer.

VESUSMT403: NUMERICAL METHODS

Unit I. Solution of Algebraic and Transcendental Equations (15L)

1. Measures of Errors: Relative, absolute and percentage errors, Accuracy and precision: Accuracy to n decimal places, accuracy to n significant digits or significant figures, Rounding and Chopping of a number, Types of Errors: Inherent error, Round-off error and Truncation error.
2. Iteration methods based on first degree equation: Newton-Raphson method. Secant method. Regula-Falsi method.
   Derivations and geometrical interpretation and rate of convergence of all above methods to be covered.
3. General Iteration method: Fixed point iteration method.

Unit II. Interpolation, Curve fitting, Numerical Integration(15L)

1. Interpolation: Lagrange's Interpolation. Finite difference operators: Forward Difference operator, Backward Difference operator. Shift operator. Newton's forward difference interpolation formula. Newton's backward difference interpolation formula.
   Derivations of all above methods to be covered.
2. Curve fitting: linear curve fitting. Quadratic curve fitting.
3. Numerical Integration: Trapezoidal Rule. Simpson's 1/3 rd Rule. Simpson's 3/8th Rule.
   Derivations all the above three rules to be covered.

Unit III. Solution Linear Systems of Equations, Eigenvalue problems(15L)

1. Linear Systems of Equations: LU Decomposition Method (Dolittle's Method and Crout's Method). Gauss-Seidel Iterative method.
2. Eigenvalue problems: Jacobi's method for symmetric matrices. Rutishauser method for arbitrary matrices.

Reference Books:

1. Kendall E. and Atkinson; An Introduction to Numerical Analysis; Wiley.
2. M. K. Jain, S. R. K. Iyengar and R. K. Jain; Numerical Methods for Scientific and Engineering Computation; New Age International Publications.
3. S. Sastry; Introductory methods of Numerical Analysis; PHI Learning.
4. An introduction to Scilab-Cse iitb.

Additional Reference Books

1. S.D. Comte and Carl de Boor; Elementary Numerical Analysis, An algorithmic approach; McGrawHillll International Book Company.

2. Hildebrand F.B.; Introduction to Numerical Analysis; Dover Publication, NY.
3. Scarborough James B.; Numerical Mathematical Analysis; Oxford University Press, New Delhi.

VESUSMTP04: Practicals

Suggested Practicals for VESUSMTP401

1. Limits and continuity of scalar fields and vector fields, using ”definition and otherwise“ , iterated limits.
2. Computing directional derivatives, partial derivatives and mean value theorem of scalar fields.
3. Differentiability of scalar field,Total derivative, gradient, level sets and tangent planes.
4. Chain rule, higher order derivatives and mixed partial derivatives of scalar fields.
5. Maximum and minimum rate of change of scalar fields. Taylor's Theorem. Finding Hessian/Jacobean matrix. Differentiation of a vector field at a point. Chain Rule for vector fields.
6. Finding maxima, minima and saddle points. Second derivative test for extrema of functions of two variables and method of Lagrange multipliers.
7. Miscellaneous Theoretical Questions based on full paper.

Suggested Practicals for VESUSMTP402

1. Linear transformation, Kernel, Rank-Nullity Theorem.
2. Linear Isomorphism, Matrix associated with Linear transformations.
3. Inner product and properties, Projection, Orthogonal complements.
4. Orthogonal, orthonormal sets, Gram-Schmidt orthogonalisation
5. Eigenvalues, Eigenvectors, Characteristic polynomial. Applications of Cayley Hamilton Theorem.
6. Diagonalisation of matrix, orthogonal diagonalisation of symmetric matrix and application to quadratic form.
7. Miscellaneous Theoretical Questions based on full paper.

Suggested Practicals for VESUSMTP403

The Practical no. 1 to 6 should be performed either using non-programable scientific calculators or by using the software Scilab.

1. Newton-Raphson method, Secant method.
2. Regula-Falsi method, Iteration Method..
3. Interpolating polynomial by Lagrange's Interpolation, Newton forward and backward difference Interpolation.
4. Curve fitting, Trapezoidal Rule, Simpson's 1/3rd Rule, Simpson's 3/8th Rule.
5. LU decomposition method, Gauss-Seidel Interative method.
6. Jacobi's method, Rutishauser method..
7. Miscellaneous theoretical questions from all units.

Scheme of Examination (75:25)

The performance of the learners shall be evaluated into two parts. The learner's performance shall be assessed by Internal Assessment with 25 percent marks in the first part and by conducting the Semester End Examinations with 75 percent marks in the second part. The allocation of marks for the Internal Assessment and Semester End Examinations are as shown below:-

I. Internal Evaluation of 25 marks

(i) One class test of 10 marks (on any type of objective questions).
(ii) One assignment of 10 marks (Different Problems to each group consisting not more than 2 students).
(iii) 5 marks for active participation.

II. Semester End Theory Examinations: There will be a Semester end external Theory examination of 75 marks for each of the courses VESUSMT301, VESUSMT302, VESUSMT303 of Semester III and VESUSMT401, VESUSMT402, VESUSMT403 of Semester IV.

1. Duration: The examinations shall be of 2 and $\frac{1}{2}$ hours duration.
2. Theory Question Paper Pattern:
   a) There shall be FOUR questions. The first three questions Q1, Q2, Q3 shall be of 20 marks, each based on the units I, II, III respectively. The question Q4 shall be of 15 marks based on the entire syllabus.
   b) All the questions shall be compulsory. The questions Q1, Q2, Q3, Q4 shall have internal choices within the questions. Including the choices, the marks for each question shall be 25-27.
   c) The questions Q1, Q2, Q3, Q4 may be subdivided into sub-questions as a, b, c, d & e, etc and the allocation of marks depends on the weightage of the topic.

III. Semester End Examinations Practicals:

At the end of the Semesters III and IV Practical examinations of three hours duration and 150 marks shall be conducted for the courses VESUSMTP03, VESUSMTP04.

In semester III, the Practical examinations for VESUSMT301, VESUSMT302 and VESUSMT303 are held together.

In semester IV, the Practical examinations forVESUSMT401, VESUSMT402 and VESUSMT403 are held together.

Paper pattern: The question paper shall have two parts A and B.
Each part shall have two Sections.

Section I Objective in nature: Attempt any Eight out of Twelve multiple choice questions (04 objective questions from each unit) ($8 \times 3 = 24$ Marks).

Section II Problems: Attempt any Two out of Three (01 descriptive question from each unit) ($8 \times 2 = 16$ Marks).

Practical Course	Part A	Part B	Part C	Marks out of	Duration
VESUSMTP03	Questions from VESUSMT301	Questions from VESUSMT302	Questions from VESUSMT303	120	3 hours
VESUSMTP04	Questions from VESUSMT401	Questions from VESUSMT402	Questions from VESUSMT403	120	3 hours

Marks for Journal and Viva:

1. Journal: 15 marks (5 marks for each journal).
2. Viva: 15 marks (5 marks each course).

Each Practical of every course of Semester III and IV shall contain at least 10 objective questions and at least 6 descriptive questions to be written in the journal.
A student must have a certified journal before appearing for the practical examination. In case a student does not posses a certified journal he/she will be evaluated for 120 marks. He/she is not qualified for Journal + Viva marks.