



# **Vivekanand Education Society's College of Arts, Science and Commerce**

**(Autonomous)**

**Sindhi Society, Chembur, Mumbai, Maharashtra – 400 071.**

*Accredited by NAAC "A Grade" in 3<sup>rd</sup> Cycle - 2017  
Best College Award – Urban Area, University of Mumbai (2012-13)  
Recipient of FIST Grant (DST) and STAR College Grant (DBT)*

**Affiliated to the  
University of Mumbai**

***Syllabus for***

**Program: B.Sc. (Mathematics)  
(Program code: VESUSMT)**

**As per Choice Based Semester and Grading System (CBSGS)  
with effect from Academic Year 2022 - 2023**

## **Program Outcomes (PO):**

A learner completing B.Sc.(Mathematics) will be able to:

- PO1 Demonstrate analytical skills in applying appropriate science principles and methodologies to solve a wide range of problems.
- PO2 Design, carry out experiments and analyze results by accounting uncertainties in different quantities measured using various scientific instruments.
- PO3 Demonstrate professional behavior of being unbiased, and truthful in all aspects of work as an individual as well as team.
- PO4 Ability to communicate science effectively by written, computational and graphic means .

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## **Program Specific Outcomes (PSO's)**

On completion of B.Sc. (Mathematics) program, learners will be enriched with knowledge and be able to

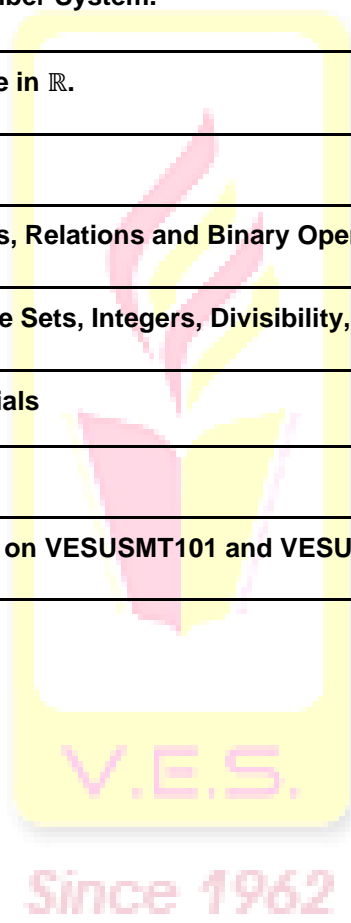
- PSO1 Think in a critical manner.
- PSO2 Know when there is a need for information, to be able to identify, locate, evaluate, and effectively use that information for the issue or problem at hand.
- PSO3 Formulate and develop mathematical arguments in a logical manner.
- PSO4 Acquire good knowledge and understanding in advanced areas of mathematics chosen by the student from the given courses.
- PSO5 Understand, formulate and use quantitative models arising in social science, business and other contexts.

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## F.Y.B.Sc. (Mathematics)

### (SEMESTER I)

CALCULUS I				
Course Code	UNIT	TOPICS	Credits	L / Week
VESUSMT101	I	First order First degree Differential equations.	2	3
	II	Real Number System.		
	III	Sequence in $\mathbb{R}$ .		
ALGEBRA I				
VESUSMT102	I	Functions, Relations and Binary Operations	2	3
	II	Countable Sets, Integers, Divisibility,		
	III	Polynomials		
PRACTICALS				
VESUSMTP101	Practicals based on VESUSMT101 and VESUSMT102		2	2



## (SEMESTER II)

CALCULAS II				
Course Code	UNIT	TOPICS	Credits	L / Week
VESUSMT201	I	Limits and Continuity.	2	3
	II	Differentiability of functions.		
	III	Applications of Derivative.		
DISCRETE MATHEMATICS				
VESUSMT202	I	Preliminary Countings	2	3
	II	Advance Countings		
	II	Permutations and Recurrence relations.		
PRACTICAL				
VESUSMTP201	Practicals based on VESUSMT201 and VESUSMT202		2	2

### Objectives:

- (1) Give the students a sufficient knowledge of fundamental principles, methods and a clear perception of innumerable power of mathematical ideas and tools and know how to use them by modeling, solving and interpreting.
- (2) reflecting the broad nature of the subject and developing mathematical tools for continuing further study in various fields of science.
- (3) Enhancing students' overall development and to equip them with mathematical modeling abilities, problem solving skills, creative talent and power of communication necessary for various kinds of employment.
- (4) A student should get adequate exposure to global and local concerns that explore them many aspects of Mathematical Sciences.

### Learning Outcomes:

1. Calculus (Sem I & II): The portion on first order, first degree differentials prepares learner to get solutions of so many kinds of problems in all subjects of Science and also prepares learner for further studies of differential equations and related fields. This course gives introduction to basic concepts of Analysis with rigor and prepares students to study further courses in Analysis. Formal proofs are given lot of emphasis in this course which also enhances understanding of the subject of Mathematics as a whole.
2. Algebra I (Sem I) & Discrete Mathematics (Sem II): This course gives expositions to number systems (Natural numbers and integers), like divisibility and prime numbers

and their properties. These topics later find use in advanced subjects like cryptography and its uses in cyber security and such related fields.

Teaching Pattern for Semester I

- (1) Three lectures per week per course.
- (2) One Practical per week per batch of one lecture for each of the courses VESUSMT101 and VESUSMT102.

Teaching Pattern for Semester II

- (3) Three lectures per week per course.
- (4) One Practical per week per batch of one lecture for each of the courses VESUSMT201 and VESUSMT202.

**Semester I**  
VESUSMT101 : CALCULUS I

**Note:** All topics have to be covered with proofs in details (unless mentioned otherwise) and examples.

**Unit 1: First order First degree Differential equations. (15 lectures).**

Review of Definition of differential equation, order, degree, ordinary differential equation and partial differential equation, linear and nonlinear ODE. Solution of homogeneous differential equations of first order and first degree. Notion of partial derivatives.

- (1) Exact equations: General solution of Exact differential equations of first order and first degree. Necessary and sufficient conditions for  $Mdx + Ndy = 0$  to be exact. Non-exact equations: Rules for finding integrating factors (without proof) for non exact equations, such as:

- i)  $\frac{1}{Mx + Ny}$  is an I. F. if  $Mx + Ny \neq 0$  and  $Mdx + Ndy = 0$  is homogeneous.
- ii)  $\frac{1}{Mx - Ny}$  is an I. F. if  $Mx - Ny \neq 0$  and  $Mdx + Ndy = 0$  is of the form  $f_1(x, y)ydx + f_2(x, y)xdy = 0$ .
- iii)  $e^{\int f(x)dx}$  ( resp  $e^{\int g(y)dy}$  ) is an I. F. if  $N \neq 0$  ( resp  $M \neq 0$  ) and  $\frac{1}{N}(\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x})$  ( resp  $\frac{1}{M}(\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x})$  ) is a function of x ( resp of y ) alone.
- iv) Linear and reducible linear equations of first order, finding solutions of first order differential equations of the type for applications to orthogonal trajectories, population growth and finding the current at a given time.

3. Reduction of order:

- (i) If the differential equation does not contain only the original function y, that is equation of type  $F(x, y', y'') = 0$ .
- (ii) If the differential equation does not contain the independent variable x that is, equation of the type  $F(y, y', y'') = 0$

## Unit 2: Real Number System (15 lectures)

- (1) Real Number System  $\mathbb{R}$  and order properties of  $\mathbb{R}$ , absolute value  $| \cdot |$  and its properties.
- (2) AM-GM inequality, Cauchy-Schwarz inequality, Intervals and neighborhoods, interior points, limit point, Hausdorff property.
- (3) Bounded sets, statements of l.u.b. axiom and its consequences, supremum and infimum, maximum and minimum, Archimedian property and its applications, density of rationals,

## Unit 3: Sequences in $\mathbb{R}$ (15 lectures)

- (1) Definition of sequence and examples, Convergence of sequences, every convergent sequence is bounded. Limit of convergent sequence and uniqueness of limit, Divergent sequences.
- (2) Convergence of standard sequences like  $\left(\frac{1}{1+na}\right) \forall a > 0, (b^n) \forall b, 0 < b < 1, (c^{\frac{1}{n}}), \forall c > 0$  and  $(n^{\frac{1}{n}})$ .
- (3) Algebra of convergent sequences, sandwich theorem, monotone sequences, monotone convergence theorem and consequences as convergence of  $\left(1 + \frac{1}{n}\right)^n$ .
- (4) Definition of subsequence, subsequence of a convergent sequence is convergent and converges to the same limit, definition of a Cauchy sequence, every convergent sequence is a Cauchy sequence and conversely.

## Reference Books:

1. R. G. Bartle – D. R. Sherbert, Introduction to Real Analysis, John Wiley & Sons, 1994.
2. R. R. Goldberg, Methods of Real Analysis, Oxford and IBH, 1964.
3. K. G. Binmore, Mathematical Analysis, Cambridge University Press, 1982.
4. Sudhir Ghorpade and Balmohan Limay, A course in Calculus and Real Analysis, Springer International Ltd, 2000.
5. G. F. Simmons, Differential Equations with Applications and Historical Notes, McGraw Hill, 1972.
6. E. A. Coddington, An Introduction to Ordinary Differential Equations, Prentice Hall, 1961.
7. W. E. Boyce, R. C. DiPrima, Elementary Differential Equations and Boundary Value Problems, Wiley, 2013.

## Additional Reference Books

1. T. M. Apostol, Calculus Volume I, Wiley & Sons (Asia) Pvt. Ltd.
2. Richard Courant – Fritz John, An Introduction to Calculus and Analysis, Volume I, Springer.
3. Ajit Kumar and S. Kumaresan, A Basic Course in Real Analysis, CRC Press, 2014.
4. James Stewart, Calculus, Third Edition, Brooks/cole Publishing Company, 1994.
5. D. A. Murray, Introductory Course in Differential Equations, Longmans, Green and Co. 1897.
6. A. R. Forsyth, A Treatise on Differential Equations, MacMillan and Co. 1956.

**Prerequisite:**

Set Theory: Set, subset, union and intersection of two sets, empty set, universal set, complement of a set, DeMorgan's laws, Cartesian product of two sets, Relations, Permutations  ${}^n P_r$  and Combination  ${}^n C_r$ .

Complex numbers: Addition and multiplication of complex numbers, modulus, amplitude and conjugate of a complex number.

**Unit I: Functions, Relations, Binary Operations & Mathematical Induction (15 Lectures)**

- (1) Definition of relation and function, domain, co-domain and range of a function, composite functions, examples, Direct image  $f(A)$  and inverse image  $f^{-1}(B)$  for a function  $f$ , injective, surjective, bijective functions, Composite of injective, surjective, bijective functions when defined, invertible functions, bijective functions are invertible and conversely, examples of functions including constant, identity, projection, inclusion, Binary operation as a function, properties, examples.
- (2) Equivalence relation, Equivalence classes, properties such as two equivalence classes are either identical or disjoint, Definition of partition, every partition gives an equivalence relation and vice versa.
- (3) Statement of well ordering property of non-negative integers, Principle of finite induction (first and second) as a consequence of WOP.

**Unit II: Integers and Divisibility (15 Lectures)**

- (1) Finite and infinite sets, countable and uncountable sets, examples such as  $\mathbb{N}, \mathbb{Z}, \mathbb{N} \times \mathbb{N}, \mathbb{Q}, (0,1), \mathbb{R}$ .
- (2) Divisibility in integers, division algorithm, greatest common divisor (g.c.d.) and least common multiple (l.c.m.) of two non zero integers, basic properties of g.c.d. such as existence and uniqueness of g.c.d. of two non zero integers  $a$  and  $b$  and that the g.c.d. can be expressed as  $ma+nb$  for some integers  $m, n$ . Euclidean algorithm.
- (3) Primes, Euclid's lemma, Fundamental Theorem of arithmetic, The set of primes is infinite, there are arbitrarily large gaps between primes, there exists infinitely many primes of the form  $4n - 1$  and of the form  $6n - 1$ .

**Unit III: Polynomials (15 Lectures)**

- (1) Congruence, definition and elementary properties, Results about linear congruence equations. Examples, Congruence is an equivalence relation on the set of integers, Residue classes and partition of  $\mathbb{Z}$ , Addition modulo  $n$ , Multiplication modulo  $n$ .
- (2) Definition of a polynomial, polynomials over  $F$  where  $F = \mathbb{Q}, \mathbb{R}$  or  $\mathbb{C}$ , Algebra of polynomials, degree of polynomial, basic properties.
- (3) Division algorithm in  $F[X]$  (without proof), g.c.d. of two polynomials and its basic properties, Euclidean algorithm (proof of the above results may be given only in the case of  $\mathbb{Q}[X]$  with a remark that the results as well as the proofs remain valid in the case of  $\mathbb{R}[X]$  and  $\mathbb{C}[X]$ ).
- (4) Roots of polynomial, relation between roots and coefficients, multiplicity of a root, Elementary consequences such as the following:
  - (i) Remainder theorem, Factor theorem.
  - (ii) A polynomial of degree  $n$  has at most  $n$  roots.
  - (iii) Complex and non-real roots of a polynomial in  $\mathbb{R}[X]$  occur in conjugate pairs. (Emphasis on examples and problems in polynomials with real coefficients).
- (5) Necessary condition for a rational number  $\frac{p}{q}$  to be a root of a polynomial with integer

coefficients (viz.  $p$  divides the constant and  $q$  divides the leading coefficient), corollary for monic polynomials (viz. a rational root of monic polynomial with integer coefficients is necessarily an integer). Simple consequences such as the irrationality is necessarily of  $\sqrt{p}$  for any prime number  $p$ . Irreducible polynomials in  $\mathbb{Q}[X]$ , Unique Factorization Theorem. Examples.

#### Reference Books:

1. David M. Burton, Elementary Number Theory, Seventh Edition, McGraw Hill Education (India) Private Ltd.
2. Norman L. Biggs, Discrete Mathematics, Revised Edition, Clarendon Press, Oxford, 1989.

#### Additional Reference Books:

1. I. Niven and S. Zuckerman, Introduction to the theory of numbers, Third Edition, Wiley Eastern, New Delhi, 1972.
2. G. Birkoff and S. Maclane, A Survey of Modern Algebra, Third Edition, Mac Millan, New York, 1965.
3. N. S. Gopalkrishnan, University Algebra, New Age International Ltd. Reprint 2013.
4. I. N. Herstein, Topics in Algebra, John wiley, 2006.

PRACTICALS  
VESUSMTP101  
Calculus I

- (1) Solving exact and non-exact, linear, reducible to linear differential equations.
- (2) Reduction of order of Differential Equations, Applications of Differential equations.
- (3) Algebraic and Order Properties of Real Numbers and Inequalities.
- (4) Hausdorff Property and LUB Axiom of  $\mathbb{R}$ , Archimedian Property.
- (5) Convergence and divergence of sequences, bounded sequences, Sandwich Theorem.
- (6) Cauchy sequences, monotonic sequences, non-monotonic sequences.
- (7) Miscellaneous Theoretical Questions based on full paper.

Algebra I

- (1) Functions, Bijective and Invertible function Composite of functions, Binary Operations.
- (2) Equivalence Relations, Partition and Equivalence classes, Mathematical induction.
- (3) Division Algorithm, Euclidean algorithm in  $\mathbb{Z}$ , Examples on expressing the gcd of two non-zero integers  $a$  and  $b$  as  $ma + nb$  for some  $m, n \in \mathbb{Z}$ .
- (4) Primes and the Fundamental theorem of Arithmetic, Euclid's lemma, there exists infinitely many primes of the form  $4n - 1$  and of the form  $6n - 1$ .
- (5) Polynomial (I)
- (6) Polynomial (II)
- (7) Miscellaneous Theoretical Questions based on full paper.



**Semester II**  
**VESUSMT201 : CALCULUS II**

**Unit-I: Limits and Continuity (15 Lectures)**

{Brief review: Domain and range of a function, injective function, surjective function, bijective function, composite of two functions (when defined), Inverse of a bijective function. Graphs of some standard functions such as  $|x|$ ,  $e^x$ ,  $\log x$ ,  $ax^2+bx+c$ ,  $\frac{1}{x}$ ,  $x^n$   $n \geq 3$ ),  $\sin x$ ,  $\cos x$ ,  $\tan x$ ,  $\sin\left(\frac{1}{x}\right)$ ,  $x^2 \sin\left(\frac{1}{x}\right)$  over suitable intervals of  $\mathbb{R}$ . No direct questions to be added.}

- (1)  $\epsilon - \delta$  definition of Limit of a function, uniqueness of limit if it exists, algebra of limits, limits of composite function, sandwich theorem, left-hand-limit  $\lim_{x \rightarrow a^-} f(x)$ , right-hand-limit  $\lim_{x \rightarrow a^+} f(x)$ , non-existence of limits,  $\lim_{x \rightarrow -\infty} f(x)$ ,  $\lim_{x \rightarrow \infty} f(x)$  and  $\lim_{x \rightarrow a} f(x) = \pm\infty$ .
- (2) Continuous functions: Continuity of a real valued function at a point and on a set using  $\epsilon - \delta$  definition, examples, Continuity of a real valued function at end points of domain using  $\epsilon - \delta$  definition,  $f$  is continuous at  $a$  if and only if  $\lim_{x \rightarrow a} f(x)$  exists and equals to  $f(a)$ , Sequential continuity, Algebra of continuous functions, discontinuous functions, examples of removable and essential discontinuity.
- (3) Intermediate Value theorem and its applications, Bolzano-Weierstrass theorem (statement only): A continuous function on a closed and bounded interval is bounded and attains its bounds.

**Unit-II: Differentiability of functions (15 Lectures)**

- (1) Differentiation of real valued function of one variable: Definition of differentiability of a function at a point of an open interval, examples of differentiable and non differentiable functions, differentiable functions are continuous but not conversely, algebra of differentiable functions.
- (2) Chain rule, Higher order derivatives, Leibniz rule, Derivative of inverse functions, Implicit differentiation (only examples)

**Unit-III: Applications of differentiability (15 Lectures)**

- (1) Rolle's Theorem, Lagrange's and Cauchy's Mean Value Theorems, applications and examples, Monotone increasing and decreasing functions, examples.
- (2) L-Hospital rule (without proof), examples of indeterminate forms, Taylor's theorem with Lagrange's form of remainder with proof, Taylor polynomial and applications.
- (3) Definition of critical point, local maximum/minimum, necessary condition, stationary points, second derivative test, examples, concave/convex functions, point of inflection.
- (4) Sketching of graphs of functions using properties.

**Reference Books:**

1. Thomas G. B., Finney R. L., & Weir M. D., Calculus and Analytic Geometry, 1996.
2. R. R. Goldberg, Methods of Real Analysis, Oxford and IBH 1964.
3. James Stewart, Calculus, Third Edition, Brooks/Cole Publishing company, 1994.
4. T. M. Apostol, Calculus Vol -I, Wiley and Sons (Asia) Pvt. Ltd.

### Additional Reference:

1. Richard Courant and Fritz John, A Introduction to Calculus and Analysis, Volume-I, Springer.
2. Ajit Kumar and S. Kumaresan, A Basic course in Real Analysis, CRC Press, 2014.
3. K. G. Binmore, Mathematical Analysis, Cambridge University Press, 1982.
4. G. B. Thomas, Calculus, 12th Edition 2009

## VESUSMT202 : DISCRETE MATHEMATICS

### Unit I: Preliminary Counting (15 Lectures)

- (1) Addition and multiplication Principle, counting sets of pairs, two ways counting.
- (2) Sterling Numbers of second kind  $S(n, k)$  or  $S_n^k$ , Simple recursion formulae satisfied by  $S(n, k)$  for  $k = 1, 2, \dots, n$ .
- (3) Pigeonhole principle simple and strong form and examples, its applications to geometry.
- (4) Permutation and combination of sets and multi-sets, circular permutations, emphasis on solving problems.

### Unit II: Advanced Counting (15 Lectures)

- (1) Binomial and Multinomial Theorem, Pascal identity, examples of standard identities such as the following with emphasis on combinatorial proofs:

$$\begin{aligned} \bullet \sum_{k=0}^r \binom{m}{k} \binom{n}{r-k} &= \binom{m+n}{r} & \bullet \sum_{i=0}^k \binom{k}{i}^2 &= \binom{2k}{k} \\ \bullet \sum_{i=r}^n \binom{i}{r} &= \binom{n+1}{r+1} & \bullet \sum_{i=0}^n \binom{n}{i} &= 2^n \end{aligned}$$

- (2) Non-negative integer solutions of equation  $x_1 + x_2 + \dots + x_k = n$ .
- (3) Principal of inclusion and exclusion, its applications, derangements, explicit formula for  $D_n$ , deriving formula for Euler's function  $\phi(n)$ .

### Unit III: Permutations and Recurrence relation (15 lectures)

- (1) Permutation of objects,  $S_n$ , composition of permutations, results such as every permutation is a product of disjoint cycles, every cycle is a product of transpositions, signature of a permutation, even and odd permutations, cardinality of  $S_n$ ,  $A_n$ .
- (2) Recurrence Relations, definition of homogeneous, non-homogeneous, linear, non-linear recurrence relation, obtaining recurrence relations of Tower of Hanoi, Fibonacci sequence, etc. in counting problems, solving homogeneous as well as non homogeneous recurrence relations by using iterative methods, solving a homogeneous recurrence relation of second degree using algebraic method proving the necessary result.

### Recommended Books:

1. Norman Biggs, Discrete Mathematics, Oxford University Press.
2. Richard Brualdi, Introductory Combinatorics, John Wiley and sons.
3. V. Krishnamurthy, Combinatorics-Theory and Applications, Affiliated East West Press.
4. Discrete Mathematics and its Applications, Tata McGraw Hills.
5. Schaum's outline series, Discrete mathematics,
6. Allen Tucker, Applied Combinatorics, John Wiley and Sons.
7. Sharad Sane, Combinatorial Techniques, Springer.

### VESUSMTP201 Calculus II

- (1) Limit of a function and Sandwich theorem, Continuous and discontinuous function.
- (2) Algebra of limits and continuous functions, Intermediate Value theorem, Bolzano-Weierstrass theorem.
- (3) Properties of differentiable functions, derivatives of inverse functions and implicit functions.
- (4) Higher order derivatives, Leibnitz Rule.
- (5) Mean value theorems and its applications, L'Hospital's Rule, Increasing and Decreasing functions.
- (6) Extreme values, Taylor's Theorem and Curve Sketching.
- (7) Miscellaneous Theoretical Questions based on full paper.

### Discrete Mathematics

- (1) Counting principles, Two way counting.
- (2) Stirling numbers of second kind, Pigeon hole principle.
- (3) Multinomial theorem, identities, permutation and combination of multi-set.
- (4) Inclusion-Exclusion principle. Euler phi function.
- (5) Composition of permutations, signature of permutation, inverse of permutation.
- (6) Recurrence relation.
- (7) Miscellaneous Theoretical Questions based on full paper.

## Scheme of Examination (75:25)

The performance of the learners shall be evaluated into two parts. The learner's performance shall be assessed by Internal Assessment with 25 percent marks in the first part and by conducting the Semester End Examinations with 75 percent marks in the second part. The allocation of marks for the Internal Assessment and Semester End Examinations are as shown below:-

### Internal Evaluation of 25 marks

- (i) One class test of 10 marks (on any type of objective questions).
- (ii) One assignment of 10 marks (Different Problems to each group consisting not more than 2 students).
- (iii) 5 marks for active participation.

**II. Semester End Theory Examinations:** There will be a Semester end external Theory examination of 75 marks for each of the courses VESUSMT101, VESUSMT102 OF Semester I and VESUSMT201, VESUSMT202 OF Semester II.

1. **Duration:** The examinations shall be of 2 and  $\frac{1}{2}$  hours duration.

2. **Theory Question Paper Pattern:**

- a) There shall be FOUR questions. The first three questions Q1, Q2, Q3 shall be of 20 marks, each based on the units I, II, III respectively. The question Q4 shall be of 15 marks based on the entire syllabus.
- b) All the questions shall be compulsory. The questions Q1, Q2, Q3, Q4 shall have internal choices within the questions. Including the choices, the marks for each question shall be 25-27.
- c) The questions Q1, Q2, Q3, Q4 may be subdivided into sub-questions as a, b, c, d & e, etc and the allocation of marks depends on the weightage of the topic.

### **III. Semester End Examinations Practicals:**

At the end of the Semesters I and II Practical examinations of three hours duration and 100 marks shall be conducted for the courses VESUSMTP101, VESUSMTP201.

In semester I, the Practical examinations for VESUSMT101 and VESUSMT102 are held together.

In semester II, the Practical examinations for VESUSMT201 and VESUSMT202 are held together.

**Paper pattern:** The question paper shall have two parts A and B.

Each part shall have two Sections.

**Section I** Objective in nature: Attempt any Eight out of Twelve multiple choice questions ( 04 objective questions from each unit) ( $8 \times 3 = 24$  Marks).

**Section II** Problems: Attempt any Two out of Three ( 01 descriptive question from each unit) ( $8 \times 2 = 16$  Marks).

Practical Course	Part A	Part B	Marks out of	Duration
VESUSMTP101	Questions from VESUSMT101	Questions from VESUSMT102	80	3 hours
VESUSMTP201	Questions from VESUSMT201	Questions from VESUSMT202	80	3 hours

1. Journal: 10 marks (5 marks for each journal).

2. Viva: 10 marks.

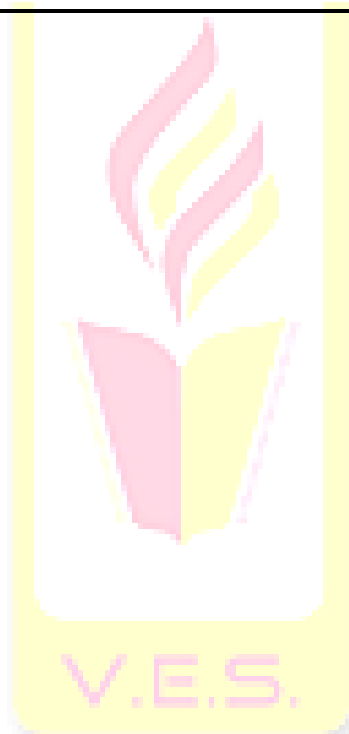
Each Practical of every course of Semester I and II shall contain at least 10 objective questions and at least 6 descriptive questions.

A student must have a certified journal before appearing for the practical examination.

In case a student does not possess a certified journal he/she will be evaluated for 80 marks.

He/she is not qualified for Journal + Viva marks.

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