Vivekanand Education Society's

College of Arts, Science and Commerce

(Autonomous)

Sindhi Society, Chembur, Mumbai, Maharashtra – 400 071.

Accredited by NAAC "A Grade" in 3rd Cycle - 2017
Best College Award – Urban Area, University of Mumbai (2012-13)
Recipient of FIST Grant (DST) and STAR College Grant (DBT)

Affiliated to the

University of Mumbai

Syllabus for

Program: B.Sc.

(Department of Mathematics)

As per Choice Based Semester and Grading System (CBSGS)
with effect from Academic Year 2025 - 2026

MINOR

T. Y. B. Sc. (Maths) SEMESTER V

COURSE TITLE: Linear Algebra II

COURSE CODE: UMNMTS5-119 [CREDITS - 04]

Course Learning Objective
The objective of this course is to: 1. Develop a foundational understanding of determinants, inner product spaces, eigen values and eigenvectors. 2. Understand determinants and their properties, including how they help in solving some systems of linear equations. 3. Explore the interconnectedness of rank of matrix, determinant of a matrix and solutions of a system of linear equations involving the matrix. 4. Understand eigenvalues and eigenvectors, their properties. 5. Determine if a matrix is diagonalizable and apply the diagonalization procedure. 6. Learn about inner product spaces and orthogonality, which help in understanding distances, angles. 7. Understand orthogonal decomposition of an inner product space and orthogonal diagonalization of a matrix and its applications. 8. The main focus of the course is on applying concepts for problem solving.

Course Learning Outcomes
After completion of this course learner will be able to: 1. Calculate determinants and utilise them to solve systems of linear equations. 2. Solve a system of linear equations and determine their number of solutions on the basis of rank, determinant etc. 3. Find eigenvalues and eigenvectors of a square matrix. 4. Compute the characteristic polynomial and understand its role in finding eigenvalues. 5. Apply the Cayley-Hamilton theorem to solve matrix-related problems. 6. Diagonalize matrices when it is diagonalizable. 7. Define and identify inner product spaces, compute norms from inner products 8. Recognize and construct orthogonal and orthonormal sets 9. Use the Gram-Schmidt process to convert a given basis into an orthonormal basis. 10. Apply orthogonal diagonalization to quadratic forms and classify matrices as positive definite or semi-definite.

Unit	Name of the Unit	(45 L)
1	**Determinants:** Inductive definition of the determinant of a $n \times n$ matrix (e. g. in terms of expansion along the first row); Basic properties of determinants (Statements only) - algebraic properties of determinants, multilinearity and alternating properties of determinants, $\det A = \det A^T$, A square matrix A is invertible if and only if $\det A \neq 0$, Minors and cofactors, Formula for A^{-1} when $\det A \neq 0$, $\det(AB) = \det A \det B$, determinant of diagonal matrix, triangular matrix; Cramer's rule, adjoint method to find inverse of a matrix, LU Decomposition;	**(15 L)**

	Row space and the column space of a matrix as a vector space, Notion of row rank and the column rank. Statements only - Equivalence of the row rank and the column rank, Invariance of rank upon elementary row or column operations, examples of computing the rank using row reduction; Relation between the solutions of a system of non-homogeneous linear equations and the associated system of homogeneous linear equations, Necessary and sufficient condition for a system of non-homogeneous linear equations to have a solution [viz., the rank of the coefficient matrix equals the rank of the augmented matrix [A|B]] (statement only), Equivalence of statements (in which A denotes an $n \times n$ matrix) such as the following: (i) The system $Ax = b$ of non-homogeneous linear equations has a unique solution. (ii) The system $Ax = 0$ of homogeneous linear equations has no nontrivial solution. (iii) A is invertible. (iv) $\det A \neq 0$. (v) $\text{rank}(A) = n$. (Statements and problems only)	
2	**Eigenvalues and eigenvectors:** (Problem based approach) Eigenvalues and Eigenvectors of $n \times n$ real matrices, the characteristic polynomial and characteristic roots of a square real matrix, Cayley Hamilton theorem and applications, properties of characteristic polynomial: similar matrices, Invariance of the characteristic polynomial and (hence of the) eigenvalues of similar matrices, every square matrix is similar to an upper triangular matrix; Minimal polynomial of a matrix, examples like minimal polynomial of scalar matrix, diagonal matrix, similar matrix, The linear independence of eigenvectors corresponding to distinct eigenvalues of a matrix, eigenspaces, Geometric multiplicity and Algebraic multiplicity of eigenvalues of an $n \times n$ real matrix, diagonalization of an $n \times n$ matrix (statements and problems), procedure for diagonalizing a matrix, examples of non diagonalizable matrices.	**(15 L)**
3	**Inner Product Spaces and Orthogonality:** Inner Products space (over R): Definition, Examples, including the Euclidean space R^n and the space of real valued continuous functions on a closed and bounded interval. Norm associated to an inner product. Cauchy-Schwarz inequality. Triangle inequality. Angle between two vectors, orthogonality of vectors, pythagoras theorem and some geometric applications in R^2. Orthogonal sets, orthonormal sets. Gram-Schmidt orthogonalization process. Orthogonal basis and orthonormal basis for a finite dimensional inner product space.	**(15 L)**

	Orthogonal complement of any set of vectors in an inner product space, Orthogonal decomposition of an inner product space with respect to its subspace, Orthogonal projection of a vector onto a line (one dimensional subspace), Orthogonal projection of an inner product space onto its subspace. Orthogonal diagonalization of an n x n matrix, Orthogonal diagonalization of n × n real symmetric matrices, Application to real quadratic forms. Positive definite, semidefinite matrices.	
Ref:	1. Suzuki, J. (2021). Linear Algebra: An Inquiry-Based Approach (1st ed.). CRC Press. https://doi.org/10.1201/9780429284984 2. Gilbert Strang: Linear Algebra and its applications, International Student Edition. 3. S Kumaresan, Linear Algebra - A Geometric Approach, PHI Learning 4. Elementary Linear Algebra: Applications Version. H. Anton, and C. Rorres. Wiley, Eleventh edition, (2014)	
Teaching Pattern: 1. Three lectures of one hour per week. 2. One Practical (2 hours) per week per batch.		

Modality of Assessment

The performance of the learners shall be evaluated by conducting the Semester End Theory Examination of 60 Marks and Internal Examination of 40 Marks.

Overall Examination and Marks Distribution Pattern
SEMESTER V (UMNMTS5-119)

Course		Grand Total
Theory	60	100
Internal	40	

A. Theory - External examination - 60% **60 marks**

Semester End Theory Assessment
Duration - Each paper shall be of 2 hour duration.
Theory question paper pattern :-
Q1, Q2, Q3: For every unit (15 marks per unit):
Attempt any three sub-questions out of five based on each unit (5 marks each question)
Q4: Attempt any three sub-questions out of 6 (2 questions from each unit- 5 marks each)

B. Theory - Internal assessment - 40%   40 marks

Sr. No.	Evaluation type	Marks
1	Test	10
2	Assignment	10
3	Active Participation	05
4	Semester End Practical Exam with Journal (20 marks test to be converted out of 10+5 marks Journal)	15

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