Practice Questions
TYBSc Mathematics Semester 6

Paper 1

1

If $\left|\frac{z-5i}{z+5i}\right| = 1$ then $z = x + iy$ lies on

a) The real axis
b) The straight line $x = 5$
c) The straight line $y = 5$
d) A circle passing through origin.

2

The function $f(z) = |z|^2$ is

a) everywhere analytic
b) nowhere analytic
c) analytic at $z = 1$
d) analytic at $z = 0$

3

If $f(z)$ is analytic in a domain D, then

a) $f^n(z)$ exist in D
b) $f^n(z)$ does not exist in D
c) $f^n(z) = 0$ for all n in D
d) $f^n(z) = 1$ for all n in D

4

The integral $\oint_C (z - z_0)^m \, dz$ is equal to

a) 0 for $m = -1$
b) $2\pi i$ for $m = -1$
c) 2 for $m = -1$
d) π for $m = -1$

5

If $f(z)$ is analytic in a simply connected domain D, then the integral of $f(z)$ is

a) Dependent of path in D

b) Independent of path in D

c) Zero

d) $2\pi i$

6

Let C be the curve of integration, L the length of C and M any constant such that $|f(z)| \leq M$ everywhere on C, then the complex line integral is

a) $\left|\int_C f(z)\,dz\right| \leq ML$

b) $\left|\int_C f(z)\,dz\right| \leq \frac{M}{L}$

c) $\left|\int_C f(z)\,dz\right| \leq \frac{L}{M}$

d) $\left|\int_C f(z)\,dz\right| = 2\pi i\frac{M}{L}$

7

If $\exp z = 2$ then $z =$

a) 0

b) 1

c) i

d) $\ln 2 + (2n + i)\pi i$

8

Radius of convergence of the series $\sum_{n=0}^{\infty} \frac{n!}{(n)^n} z^n$ is

a) ∞

b) 1

c) e

d) $\frac{1}{e}$

9

The principal part in the Laurent's series expansion of $f(z) = \frac{1}{z(z-1)}$, $0 < |z| < 1$

Is

a) $\frac{1}{z}$

b) $-\frac{1}{z}$

c) $-\frac{1}{z^3} + \frac{1}{z^2} - \frac{1}{z}$

d) $-\frac{1}{z^2}$

10

For the function $f(z) = \frac{\sin z}{(z-3)^2}$, has pole of

a) Order 3 at $z = 2$

b) Order 2 at $z = 3$

c) Order 2 at $z = \frac{3}{\pi}$

d) Order 2 at $z = 2$

Paper 2

1. From the given list of pairs group, pick the pair of non-isomorphic groups
(a) $3\mathbb{Z}/12\mathbb{Z}$ and $\mathbb{Z}_4$ (b) $8\mathbb{Z}/48\mathbb{Z}$ and $\mathbb{Z}_6$
(c) $\mathbb{Z}_4$ and V_4 (d) $(\mathbb{Z} \times \mathbb{Z})/(2\mathbb{Z} \times 2\mathbb{Z})$ and $\mathbb{Z}_2 \times \mathbb{Z}_2$

2. Let $H_1 = \{\sigma \in S_n : \sigma(n) = n\}, H_2 = \{\sigma \in S_n : \sigma(k) = k, \text{ for some } k, \ 1 \leq k \leq n\}$. Then

   (a) H_1, H_2 are normal subgroups of S_n.

   (b) H_1 is a normal subgroup of S_n but H_2 is not a normal subgroup of S_n.

   (c) H_1, H_2 are not normal subgroups of S_n

   (d) H_2 is a normal subgroup of S_n but H_1 is not a normal subgroup of S_n.

3. Let $G = \mathbb{Z}_4 \times \mathbb{Z}_4$ and $H = \mathbb{Z}_4 \times \{\bar{0}, \bar{1}\}$, $K = < (\bar{1}, \bar{2}) >$ be subgroups of G Then
(a) G/H is isomorphic to G/K (b) G/H is isomorphic to $\mathbb{Z}_2 \times \mathbb{Z}_2$
(c) H and K are isomorphic. (d) none of these.

4. Which of the following is a subring of $\mathbb{Q}(+,\cdot)$
(i) $R = \left\{\frac{a}{b} \,; a, b \in \mathbb{Z}\, (a,b) = 1,\ b \text{ is not divisible by } 3\right\}$
(ii) $R = \left\{\frac{a}{b} \,; a, b \in \mathbb{Z}\, (a,b) = 1,\ b \neq 0, b \text{ is divisible by } 3\right\}$
(iii) $R = \{x^2 : x \in \mathbb{Q}\}$
(iv) $R = \left\{\frac{a}{b} = a, b \in \mathbb{Z}, b \neq 0 \ \ (a,b) = 1 \ \ a \text{ is divisible by } 3\right\}$
(a) (i) and (iv) (b) (ii) and (iv) (c) (i) and (ii) (d) only (i)

5. Let R be a ring and a, b be non-zero elements of R. The equation $ax = b$ has
a) a unique solution in R
b) at most one solution in R
c) may have more than one solution in R
d) None of the above

6. If $R_1 = \mathbb{Z}[\sqrt{2}] = \{a + b\sqrt{2} : a, b \in \mathbb{Z}\}, R_2 = \mathbb{Z}[\sqrt{5}] = \{a + b\sqrt{5} : a, b \in \mathbb{Z}\}, R_3 = \mathbb{Q}[\sqrt{2}] = \{a + b\sqrt{2} : a, b \in \mathbb{Q}\}, R_4 = \mathbb{Z}[i] = \{a + bi : a, b \in \mathbb{Z}\}$, then
(a) R_1, R_2, R_3, R_4 are integral domains which are not fields.
(b) R_1, R_2, R_4 are integral domains which are not fields and R_3 is a field.
(c) R_1, R_2, R_3, R_4 are all fields
(d) None of the above

7. Consider the ring $\mathbb{Z} \times \mathbb{Z}$ under component wise addition and multiplication.
Let $I = \{(a, -a) = a \in \mathbb{Z}\}$
$J = \{(a, 0) = a \in \mathbb{Z}\}$
(a) I and J are ideal of $\mathbb{Z} \times \mathbb{Z}$ (b) I and J are subrings of $\mathbb{Z} \times \mathbb{Z}$
(c) neither I nor J are ideal of $\mathbb{Z} \times \mathbb{Z}$ (d) J is a subring of $\mathbb{Z} \times \mathbb{Z}$, but I is not

8. Let $R = C[0,1]$, the ring of continuous real valued functions on $[0,1]$ under pointwise addition and multiplication, $I = \{f \in R : f(1/2) = 0\}$.
(a) I is not an ideal. (b) I is a prime ideal but not a maximal ideal.
(c) I is a maximal ideal. (d) I is an ideal but not a prime ideal.

Paper 3

Unit 1

$f, g : \mathbb{R} \longrightarrow \mathbb{R}$ are any maps, such that $f \circ g$ and $g \circ f$ are continuous (distance being usual). Then

(a) $f : \mathbb{R} \longrightarrow \mathbb{R}$ and $g : \mathbb{R} \longrightarrow \mathbb{R}$ are continuous
(b) $f \circ g = g \circ f$
(c) At least one of f and g is coninuous.
(d) Neither f nor g may be continuous.

Let (X, d) be a metric space where X is a finite set and (Y, d') be any metric space. Let $f : X \longrightarrow Y$. Then the statement which is **not true** is

(a) f is continuous on X
(b) $f(X)$ is bounded.
(c) If A is open in X, $f(A)$ is open in Y
(d) If B is closed in Y, $f^{-1}(B)$ is closed in X.

$f : \mathbb{R}^2 \to \mathbb{R}$ defined as $f(x, y) = x + |y|$, is

(a) continuous on $\mathbb{R}^2$
(b) continuous on $\mathbb{R}^2$ except at $(0, 0)$
(c) continuous only at $(0, 0)$
(d) nowhere continuous on $\mathbb{R}^2$

Let (X, d) be a metric space and $f \in C(X, \mathbb{R})$ be a bounded function. Then, f

(a) attains both its bounds.
(b) attains at least one of its bounds.
(c) may not attain either of its bounds.
(d) None of the above.

Unit 2

Let (X, d) be a connected metric space. If $f : X \longrightarrow \mathbb{R}$ (d usual) is a non-constant continuous function .Then, $f(X)$ is

(a) finite set (b) countable set. (c) singleton set (d) uncountable set.

The unit circle $S^1 = \{x \in \mathbb{R}^2 : \|x\| = 1\}$ is (distance Euclidean)

(a) Compact and Connected
(b) Compact but not Connected
(c) Connected but not Compact
(d) neither Compact nor Connected

Unit 3

The least integer value of k for which $\left\{\dfrac{e^{-nx}}{n^k}\right\}$ is uniformly convergent on $[0, \infty)$ is

(a) 0 (b) 1 (c) -1 (d) 2

If $\{f_n\}$ and $\{g_n\}$ are sequences of functions on $S, S \subseteq \mathbb{R}$ converging uniformly to f and g respectively on S then the following sequence of functions may not converge uniformly of S to the given function.

(a) $\{f_n + g_n\}$ to $f + g$. (b) $\{f_n - g_n\}$ to $f - g$. (c) $\{\lambda f_n\}$ to λf. (d) $\{f_n * g_n\}$ to $f * g$.

$-n$

Let $f_n(x) = \begin{cases} x & \text{if } x \leq n \\ n & \text{if } x > n \end{cases}$

(a) $\{f_n\}$ converges uniformly on $\mathbb{R}$ to a bounded function.
(b) $\{f_n\}$ converges uniformly on $\mathbb{R}$ to an unbounded functions.
(c) $\{f_n\}$ is not pointwise convergent on $\mathbb{R}$.
(d) $\{f_n\}$ converges pointwise on $\mathbb{R}$.

If α is a non-zero real number then the radius of convergence of $\sum\limits_{n=1}^{\infty} \alpha^n x^n$ is

(a) $|\alpha|$ (b) $\dfrac{1}{|\alpha|}$ (c) 0 (d) ∞

Paper 4

The minimum number of colors required for proper vertex coloring of a null graph on p vertices is

(a) p
(b) $p - 1$
(c) 1
(d) $2p$

The polynomial $t^4 - 4t^3 + 6t^2 - 3t$ is the chromatic polynomial of a graph G then order of G is

(a) 4
(b) 6
(c) 3
(d) 2

The edge connectivity of a graph G is

(a) nothing to do with degrees of the vertices of the graph
(b) equal to maximum degree of the graph
(c) equal to minimum degree of the graph
(d) less than the maximum degree of the graph

K_n is planar if

(a) $n > 4$
(b) $n \leq 4$
(c) $n = 5$
(d) $n = 7$

Let G be a simple undirected planar graph on 10 vertices with 15 edges, If G is connected graph, then the number of bounded faces in any embedding of G on the plane is equal to

(a) 3
(b) 4
(c) 5
(d) 6

If G is simple planar graph with $p \geq 3$ without triangle then number edges q is

(a) $= 3p - 6$
(b) $< 3p - 6$
(c) $\leq 3p - 6$
(d) $> 3p - 6$

The number distinct systems of distinct representatives of the family $A_1 = \{1,2\}, A_2 = \{2,3\}, A_3 = \{3,4\}, A_4 = \{4,5\}, A_5 = \{5,1\}$

(a) 1
(b) 2
(c) 3
(d) 5

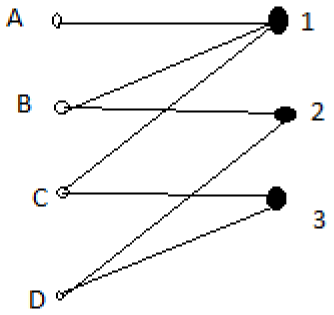
Rook polynomial for the board obtained by deleting a square from 2 × 2 chess board is

(a) $1+2x+x^2$
(b) $1+3x+2x^2$
(c) $1+x+x^2$
(d) $1+3x+x^2$

Let $f(x)$ be the generating function of a_n and $g(x)$ be the generating function of b_n. Then the generating function for the sequence a_n+b_n is given by

(a) $f(x)-g(x)$
(b) $f(x).g(x)$
(c) $f(x)+g(x)$
(d) $\sqrt{f(x)+g(x)}$

How many possible maximal matchings can be obtained from the bipartite graph below:



(a) 3
(b) 6
(c) 4
(d) 5