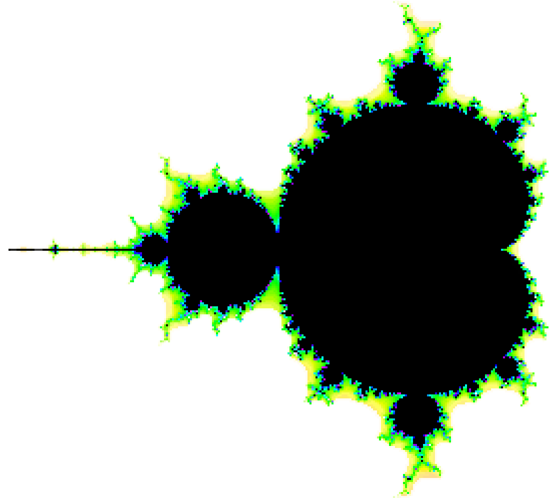

# Glimpses

# Mathemight 07

**18th, 19th & 20th January, 2013**

**Sponsored by National Academy of Sciences, India (NASI)**

Department of Mathematics
Vivekanand education society's College of Arts Science & Commerce

## Inside

1. Might of Mathematics – an account of Mathemight from the Convener

2. Glimpses into the talks/papers presented during the conference

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**Vikas Ware**

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## Report of the conference “Mathemight -7” :

I am delighted to add a few words to ‘Glimpses’ an account of the conference ‘ Mathemight-7’.

The students’ conference ‘Mathemight 7’ was organized this year at the National level with an élan, which would not have been possible without the support of NASI Mumbai chapter and the enthusiasm and guidance of Prof. Pani.

On the 18th, 19th, 20th of January 2013 students and practitioners of Mathematics gathered in the auditorium of VES College of Arts Science & Commerce to participate in Mathemight-7, the seventh conference in a series of convention, committed to encourage Mathematically inclined students .The Conference Mathemight organized by the department of Mathematics, VES college, and sponsored by National Academy of Sciences ,Mumbai Chapter , is one of the few intercollegiate math competitions held in Mumbai.

The conference witnessed a congregation of 200 highly motivated students from various Mumbai colleges and also from colleges outside the city, all eager to listen to the popular talks of eminent mathematicians and to present their own papers /posters on the theme “Surprises in Mathematics and application.”

The list of speakers in this three day conference included brilliant personalities like Prof. Rajeeva Karandikar (Director of the Institute of Mathematics, Chennai), Dr. Kamlesh Chakraborty (Deputy Governor, Reserve Bank of India), Prof. Amiya K Pani (Institute Chair Professor, Industrial Mathematics Group, IIT Mumbai), Dr.M.Vanninathan (TIFR, Bangalore),Dr. M.C.Joshi (IIT Bombay), Dr.Sharad Bhartiya (department of Chemical engineering, IIT Bombay) and Dr. Avinash Dharmadhikari (Tata Motors).

Day one began with a brief inaugural session where the genesis and theme of Mathemight were elaborated. This was followed by an enthralling key note address, titled, ‘Introduction to Cryptography’ by Prof.Karandhikar, who initiated the talk by interesting anecdotes of cryptography during world war I, and went on to explain that how inclusion of certain aspects of Number theory in the encryption algorithms strengthened the codes mightily. As a consequence, the unauthorized breaking of codes has become almost impossible in present times. To quote from his slides ‘It seems that in the World War I era, the cipher bureau in Room 40 did not have mathematicians. By the time of World War II, the team had been expanded to include Mathematicians.’

This talk was followed by the address given by the Deputy Governor of the Reserve bank of India, Dr. Chakrabarty. In his talk, ‘The Magical World of Mathematics- the charm, challenges and career prospects,’ he went on to explain the importance of Mathematics as a science and how Mathematics and Statistics can be used effectively in Banking, Finance and Economics. But he also warned the students against the improper interpretation of Math in economics and finance. In this context he raised certain thought provoking questions. At times his hilarious comments left the students in splits of laughter. To quote from his paper, ‘Mathematics is, perhaps, the oldest of sciences that has existed, developed and matured in either explicit or in latent form over thousands of years. The concept of numbers was not only known to prehistoric man but may also be known to animals. When lions hear a neighboring pride roaring, they calculate how many lions are roaring compared to the number of lions in their own pride. If there are more lions in their own pride, or the numbers are equal, or the other pride out numbers them by up to three to one, they will always roar back. If the other pride out numbers them by more than three to one, they stay quiet.'

(http://www.rbi.org.in/scripts/BS_SpeechesView.aspx?id=770 )

Highlight of the post lunch session was the presentation by students on the theme 'Surprises in Mathematics and Applications', and a poster competition on the same theme. The topics discussed ranged from Mathematical modeling, Chaos theory, Fractals, DNA, Wining a Soccer game and Poisson distribution, Vedic Mathematics to Encryption and Decryption. The day culminated with a career talk delivered by well-known career counselor Mrs. Manchandai.

Day 2 started with the talk titled, 'How to make Objects Disappear', by Dr. Vanninathan. He initiated his talk by telling some anecdotes about how from time immemorial humanity has been fascinated by the phenomena of invisibility and disappearing acts. He went on to introduce the Mathematics and Physics behind the invisibility cloak. Conclusion of the talk was the challenges and issues generated by the phenomenon of invisibility.

The major part of the post lunch session on day two was packed with students' presentations, which spilled over to lunch time as well. Students from various colleges of Mumbai and outside ranging from Mithibhai college in western suburbs, Jai hind college in the city to Ratnam in the eastern suburbs and IIT Mandi, participated in the conference. The judges and the audience showed extreme patience and interest. The students were busy showcasing their intellectual might through Mathematics to a panel of judges. Dr.Pawle from Mumbai University, Department of Mathematics and Dr Neela Nataraj from IIT Bombay, graced the chairs of the judges for the occasion.

Post lunch session on the second day also witnessed some interesting presentations by faculty members of different colleges. Papers ranged from 'Use of invariants in problem solving', 'Mathematics in Diet Plan' and 'Linear Algebra in Coding theory'.

The talk titled 'Key to key technologies and its impact on research and education in Mathematics,' by Prof. Pani clinched the second day. In his talk he emphasized that 'the demand of a maximum output of industrial research and development today can only be fulfilled by increasing use of mathematical methods.'

Although Day 3 was a Sunday, it witnessed students lining up at the auditorium of the college, to listen to the series of talks scheduled for the day. The day began with the talk titled "Motion of a Satellite in Orbit" by Prof. Mohan C Joshi, IIT Bombay. This brilliant presentation had a great impact on a large number of students, and some students decided on the spot to pursue a career in rocket science.

The next address titled 'Mathematics in Biosciences,' was delivered by Dr. Sharad Bharatiya. It was indeed very interesting to note how Mathematics is required to understand and analyze certain aspects of biological sciences. With lucid examples he explained the importance of quantified-elucidation for certain topics in bioscience. In his own words 'dramatic advances in experimental techniques and measurements in biological sciences over the past two decades have enabled us to quantify biological systems thereby opening the doors that can bring to bear the might of mathematics to unravel the mysteries of biology.' It is noteworthy that one of the student's presentation dealt with Graph theory in DNA Science.

The post lunch session ended with the talk titled, 'Mathematics in reliability Management' by Dr. Avinash Dharmadhikari. The talk was interspersed with examples and case studies. He went on to show importance of modeling and statistical tools in achieving reliability targets in the industry.

During the entire proceeding of Mathemight-7, it was indeed very encouraging that each and every Mathematician who came to address the students made their topics simple enough to be accessible and deep enough to capture the true beauty of Mathematics.

At the end of the paper presentation session judges gave their valuable comments on each presentation. The program culminated with a valedictory session and with encouraging words from Prof. Pani and the faculty members of VES College.

At the end of the three day conference the organizers and volunteers felt that their endeavors were justified and were happy to note that this exceptional branch of science is flourishing and alive amongst the young minds in the country. Indeed Mathematics invokes the creative and the imaginative part of the human mind.

In today's world of instant gratification it was heartwarming to watch students showing dedicated interest towards a pure scientific field. They truly "put all the troubles in a bracket and substituted the problems with solution."

We have presented the abstract of all these talks in the following pages of Glimpses. For detailed elucidation please visit http://www.vesasc.org/math_dept.php Mathematics. Also Glimpses records some of the presentations of the students .

Till the next Mathemight,

Dipta Dasgupta.

Convener, Mathemight -7

# Popular Series

## 1.

## Key note address

Rajeeva Karandikar (Director, Chennai Institute of Mathematics)

**Title: Introduction to Cryptography.**

**Abstract:**
In the modern internet era, secrecy of messages exchanged and digital signatures have become very important. The talk will give an overview of ideas behind this exciting application of mathematics**.**

**Excerpts from the talk:**

“It is believed that breaking Enigma had a big impact on the course of the World War II. It is less well known that an encrypted telegram sent by German Foreign Secretary Arthur Zimmermann to the German Ambassador Johann von Bernstor_ in Washington also had an impact on World WarI. The telegram was intended for German Ambassador Heinrich von Eckardt in Mexico City seeking an alliance with Mexico against United States. The encrypted telegram was intercepted by the British. The telegram has been termed as the Zimmermann telegram. The encrypted telegram was broken by Room 40, the Admiralty's cipher bureau, named after the office in which it was initially housed. The team in Room 40 consisted of linguists, classical scholars and crossword addicts. The decoded message was passed onto United States by the British. This played a major role in USA's decision to enter the WWI against Germany. It seems that in the World War I era, the cipher bureau in Room 40 did not have mathematicians. By the time of World War II, the team had been expanded to include Mathematicians. Let us examine possible reasons behind this……….”

“In a few decades after WWII, usage of computers became common and if the WWII era algorithms were still used in say the 80's, it would have been possible to break the code easily using power of a workstation. But if the hackers could use computers, so could the sender and receiver and thus use more complex algorithms.
Now instead of the alphabet for the message being A,B,C,…., the alphabet is just {0,1} and every message is coded as a string of 0's and 1's, i.e. as Binary string as it is stored on computer hard disc. When the messages were a string of alphabets, linguists had a role - in looking for patterns. If the encrypted message could be differentiated from pure gibberish that would give a starting point for cryptanalysis.
When message as well as encrypted message is a long string of 0's and 1's, role of linguists has been reduced to deciding if a given text is meaningful text in the language or not. Now

finding a pattern in a string of 0's and 1's can be thought of as follows: Can the given string be differentiated from results of a fair coin toss, with say head recorded as 1 and tail recorded as 0. Thus one necessary condition that emerges is that the output of an encryption algorithm should appear to be a random bit stream i.e. it should be indistinguishable from output of a random bit stream.

(for detailed presentation please visit http://www.vesasc.org/math_dept.php )

**2**

Dr. K.C.chakrabarty ( Deputy Governer, Reserve bank Of India)

**"The Magical World of Mathematics- the charm, challenges and career prospects."**

**Abstrac**t: In his talk, DG would dwell upon the contribution of mathematics towards the development of other sciences and its applications in the field of economics, finance and banking.

He would also talk about mathematics being a challenging discipline and the career prospects for students pursuing mathematics especially in the field of banking and finance.:

**Excerpts from his talk:**

**"The beauty and charm of mathematics has lured, intrigued and inspired countless** geniuses across the globe to spend sleepless nights in the hope of unraveling its mysteries. Why have the seekers of knowledge been attracted to mathematics from time immemorial? I feel the primary charm of mathematics is that it is both interesting and, if you can crack its intricacies, enjoyable. People like its challenge, its clarity, and the fact that in solving problems of mathematics you know when you are right. The study of mathematics can satisfy a wide range of interests and abilities. It helps develop one's imagination and aids in building a clear and logical thought process"

"Like much of engineering, financial mathematics constructively uses fundamental mathematical and scientific principles with professional practices to yield products and processes….

"With the expanding scope of business and finance, demand for mathematical acumen and empirical analyses have become ever increasing. However, we need to guard against utter predominance and capture of the finance profession by the students of mathematics….."

"Whether we deal with mechanical, electrical or electronic objects such as the light, fan, TV, car, bicycle or computers – understanding their functioning calls for use of mathematics. We all perform tasks ranging from simple arithmetic to complex computations as we deal with money, deposits, insurance, income tax, and so on……… "

“ In The Republic, the great Greek philosopher Plato presented a profound argument for why mathematics should be required for all high school and college students. He argued that mathematics and geometry teach problem-solving skills and an ability to analyze and think. It is also important to study mathematics because it gives one a different perspective on things. Learning mathematics involves a different type of thinking that is not addressed in other subjects………”

“……….. The students of mathematics must, therefore, be extremely careful as conclusions based on improper use of numbers can lead to adverse policy decisions.”

(http://www.rbi.org.in/scripts/BS_SpeechesView.aspx?id=770), http://www.vesasc.org/math_dept.php

**3**.

Dr. Vanni Nathan (***TIFR*** Centre For Applicable Mathematics, Bangalore)

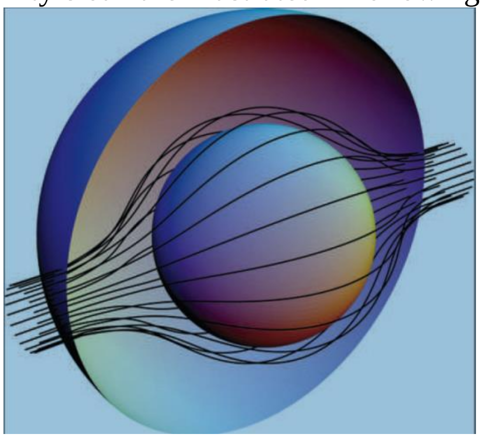
**How to make Objects Disappear?**

**Abstract**: Humanity is always fascinated by the phenomena of invisibility and disappearing acts. There are science fiction films based on these themes which attract the attention of even lay people. On the scientific front, there have been a lot of efforts (theoretical, experimental and technological) to try to achieve invisibility. The big challenging question is whether success will ever be achieved to the level of expectation. The purpose of this talk is to introduce the audience to this fascinating theme and explain the mathematics behind the phenomenon.

**Excerpts:**

“There are many, many ways in which an object can be made invisible, and some of them, as we shall see, appear to fall under the banner of magical. This is the magic of science, of course, rather than the magic of wishful thinking or the miraculous act in which something happens beyond the laws of physics and normality. Science is, indeed, a hard task master, but fortunately it has not stifl ed the free ranging nature of literary imagination − for certainly, both ancient and modern writings are rich with the ideas of magic, invisibility and the miraculous. The desire to walk unobserved through the world and to command invisibility at will is a power that many humans may have wished for, but it is a power that no one person should have restricted access to. But this being said, we now live in a world where the technology to make solid objects invisible to outside detection does exist. It is a reality, and we assuredly do live in an age of miracles and wonders. These fledgling devices will change the world and the way in which we see it or don’t see it, as the case may be………”

“The realization of a two dimensional cloaking device is exactly what David Schurig and co-workers at the Duke University and Imperial College labs announced online in their October 19,2006, article in *Science Express.* The ray paths required to realize a true three-dimensional invisibility cloak are illustrated in following Fig

…….  (image courtesy of david schurig)

Ray paths followed in a three-dimensional invisibility cloak. In this case the light rays are diverted around the small central sphere – the invisible cavity in which an object can be hidden from view. The region between the two spheres contains the all-important metamaterial substrate………”

(for detailed presentation please visit http://www.vesasc.org/math_dept.php )

4.

Amiya K.Pani (Department chair,IIT Bombay, Industrial Mathematics Group)

**key to key technologies and its impact on research & education in Mathematics.**

SKETCH OF THE TALK

- **Usefulness of mathematics as conceived by society**
- **Hope & challenges ahead**
- **Some questions**
  - Why has mathematics become so important for Industry in recent years?
  - Which kind of mathematics is needed for this Purpose?
- **Problems solving activity**
  - What are the consequences for education in Mathematics?
  - In its new avatar will it loose its beauty that is its rigour
- **some case studies**

**Excerpt:**

"The demand of a maximum output of industrial research & development today can only be fulfilled by an increasing use of mathematical methods.
Examples are simulation methods, which allow to reduce the experimental & constructive effort for the development of complex products….."

"Key words for the Role of Mathematics in Industry :-

Simulation, Data Reduction, Control, Optimization and Visualization….To provide an answer to to question 1, : "MODERN COMPUTING ALLOW THE EVALUATION OF REALISTIC MATHEMATICAL MODELS"

## 5.

Dr. Mohan Joshi (Department of Industrial Mathemayics , IIT Bombay)

**Motion of a satellite in orbit**

**Abstract**: First, we will describe a mathematical model for orbiting satellite. Subsequently, we will show how a simple notion like rank of a matrix could be used to derive whether we can control its dynamic motion in space.

## Excerpts:

" QUERY!
What happens when one of the control inputs or thrusts becomes inoperative, due to malfunctioning of rocket boosters?

CASE 1: $U_1 = 0$ reduces B to
$B = [0\ 0\ 0\ 1]^T$
This gives controllability matrix

K = [B,AB,A2B,A3B]

$$\begin{bmatrix} 0 & 0 & 2\omega & 0 \\ 0 & 2\omega & 0 & -2\omega^3 \\ 0 & 1 & 0 & -4\omega^2 \\ 1 & 0 & -4\omega^2 & 0 \end{bmatrix}$$

K has rank 4.

so the loss of radial thrust **does not destroy controllability** of satellite in motion, it is possible to maneuver satellite with radial thrust only.

CASE 2: $U_2 = 0$ and hence B reduces to

$B = [0\ 1\ 0\ 0]^T$.

So, the Kalman matrix

K = [B,AB,A2B,A3B]

$$= \begin{bmatrix} 0 & 1 & 0 & -\omega^2 \\ 1 & 0 & -\omega^2 & 0 \\ 0 & -0 & -2\omega & 0 \\ 0 & -2\omega & 0 & 2\omega^3 \end{bmatrix}$$

K has rank 3.
Hence, system is NOT CONTROLLABLE. So, loss of tangential thrust destroys maneuverability of the satellite……..”

(for detailed presentation please visit http://www.vesasc.org/math_dept.php )

## 6

**Dr. Sharad Bhartiya**

Department of Chemical Engineering, IIT Bombay

**Mathematics in Biological Sciences –**

**Abstract**:
That our society perceives biology and mathematics as orthogonal is evident from the fact that many Class XII students choose and focus either on biology or mathematics. Indeed, the intricate workings of naturally evolved biological systems are far less understood than the operation of the most complex engineered system such as a space rocket.
However, dramatic advances in experimental techniques and measurements in biological sciences over the past two decades have enabled us to quantify biological systems thereby opening the doors that can bring to bear the might of mathematics to unravel the mysteries of biology. This talk will outline a few biological systems and the corresponding mathematical formulation. Our ability to solve these problems has the potential to guide newer experiments and thus paving deeper inroads in Biology.

**Excerpts**:

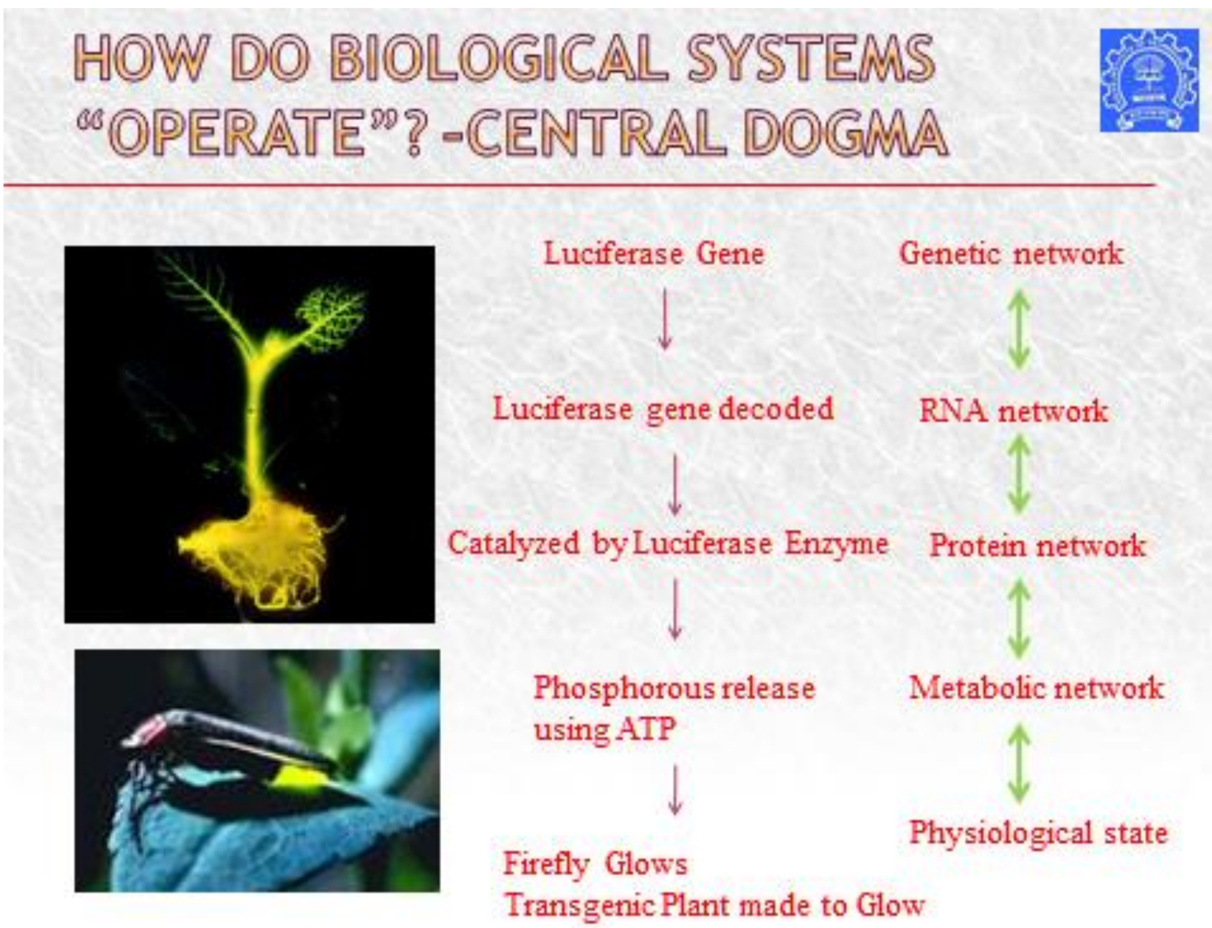
**“Quantification in Biological Sciences:**

- Historically, biology has been a descriptive science.

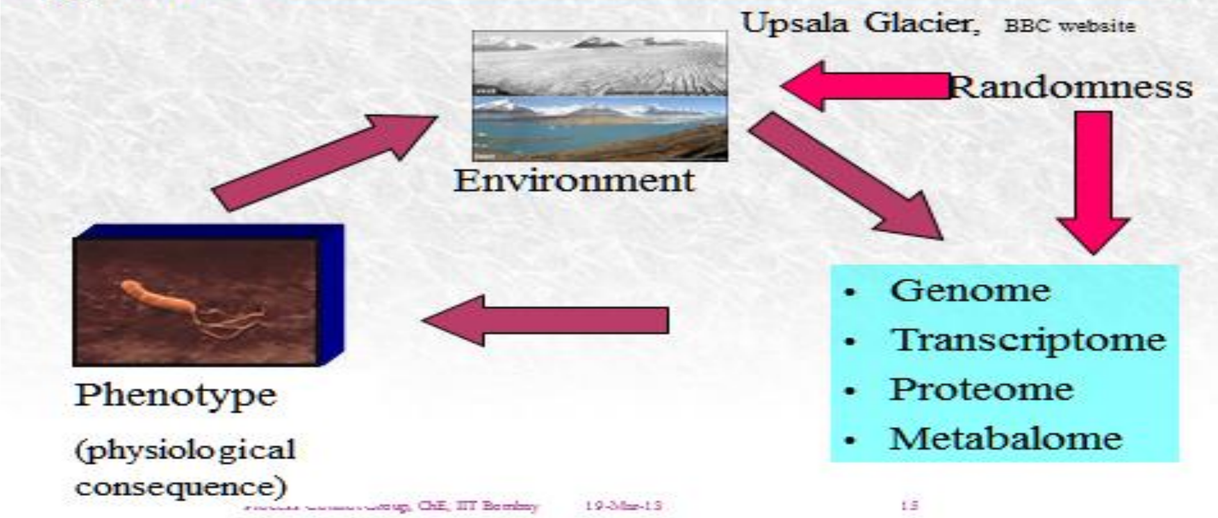
- Modern Biology has led to quantification at molecular level (sub-system).

- Similar to engineering systems that are quantified to a level that they are designed, optimized and optimally operated.
- Principles of system science can be applied to component biology: **System-wide**
- **Approach…….”**

“



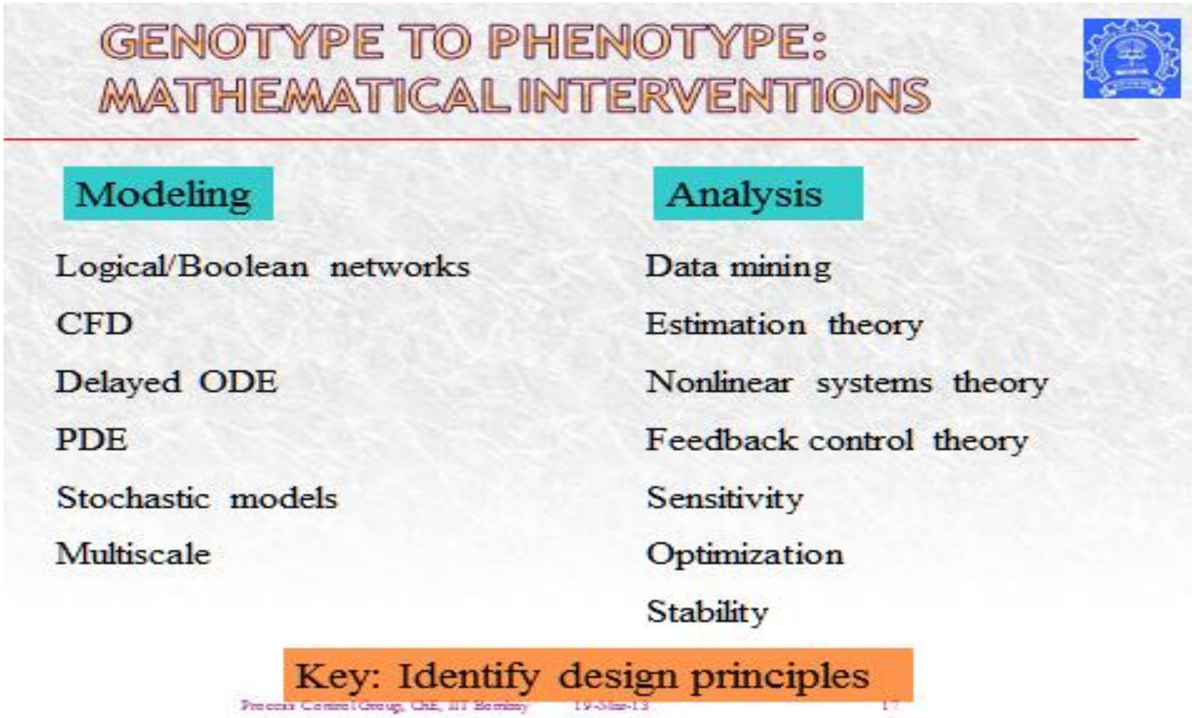
# Genotype to Phenotype: An Integrated Approach




# GENOTYPE TO PHENOTYPE

| | |
|---|---|
| Genome | Gene expression is triggered |
| Transcriptome | mRNAs are synthesized |
| Proteome | Necessary enzymes are made |
| Metabalome | Enzymes catalyze substrates |
| Phenotype | Metabolites react to trigger |

Presence of genome does not ensure a phenotype
It requires a specific state in the hierarchical chain.




**“Mathematics for biological sciences**

- “Linear Algebra – metabolic flux analysis
- Ordinary differential equations – lumped modeling
- Partial differential equations – drug delivery, metastasis
- Stochastic differential equations – heterogeneity, uncertain
- Boolean algebra – large scale networks, drug discovery
- Optimization – evolutionary biology, biotech
- Statistics – old companion of biologists
- Artificial intelligence – bioinformatics
- Lyapunov stability – perturbations before disease?”

(for detailed talk please visit http://www.vesasc.org/math_dept.php )

**7.**

Dr.Avinash Dharmadhikari

General Manager (Quality. Systems and Reliability) at *TATA MOTORS*, Pune
**Mathematics in reliability Management**

**Abstract** : This is a tutorial paper about reliability management. In an industrial set up, when a new product is launched, design engineer has to understand, what is market need and define reliability target, propose concept, design the product, allocate the reliabilities, ensure the verification and validation of the design, propose service policy, market the product and based on warranty returns revisit the design, process, or service for improvement. We take up a specific component, in an automobile, dummy data and explain major steps in this cycle. Further, we explain the role of statistical tools, like DFMEA, distribution, testing, reliability prediction, DOE, during such a decision process.

Excerpts:

**"What we want to be is the best motorcar company, being the largest is incidental."**

*****************************************************

# Presentation of Faculty:

## 1.
## Use of Invariants to solve Puzzles

Dr. Mangala R. Gurjar , Dept of Mathematics, St Xavier's College, Mumbai

**Abstract:**

Mathematical discoveries, though not intended to solve puzzles or day to day problems around us have some unexpected applications to solving puzzles or determining that they are not solvable. In this article I will give such examples which are interesting. These are based on the idea of invariants which simplify the problems dramatically.

The well known15-puzzle appeared in the United States of America in 1870. It soon became extremely popular. The craze hit Europe too. But the inventor Sam Loyd who invented the puzzle failed to get the patent. The puzzle is insolvable and at the patent office the inventor was expected to present a working model of his invention. But even though Loyd was denied the patent he "caused the world to rack its brain over a box with moving blocks" in his own words.

**Introduction** The well known15-puzzle appeared in the United States of America in 1870. It soon became extremely popular. The craze hit Europe too. But the inventor Sam Loyd who invented the puzzle failed to get the patent. The puzzle is insolvable and at the patent office the inventor was expected to present a working model of his invention. But even though Loyd was denied the patent he "caused the world to rack its brain over a box with moving blocks" in his own words.

## 2

# Applications of Linear Algebra to Coding Theory

Prof.Surjeet kaur, Dept. of Mathematics SIES College. Sion (Mumbai)

**ABSTRACT:-**
Linear Algebra is an important area of Mathematics learnt at the Undergraduate level by students graduating in Mathematics. It is at the heart of many scientific, engineering and industrial applications; general methodologies identified being extensive use of iterative solvers, structured eigenvalue problems, parameter dependent linear systems and eigenvalue problems, direct solvers etc. This paper introduces Coding Theory as an interesting application of Vector Spaces over a finite field consisting of two elements only viz {0,1} with operations of addition and multiplication defined appropriately. Coding Theory is a relatively recent application of Mathematics to information and communication systems. It has found a wide range of applications from deep space communication to quality of sound in compact discs and wireless phones. It turns out that a rich set of mathematical ideas and tools mainly from Linear Algebra can be used to design good codes. The aim of this paper is to introduce Coding Theory to students with an elementary knowledge of Linear Algebra viz Vector Spaces via a well-known class of codes known as Hamming codes. Interesting properties and projects related to Hamming codes are introduced.

## 3.

# Mathematics in Diet Plan

Mr. Laxman Naik,dept. Of Mathematics, Mithibai College(Mumbai)

**Abstract** :
Mathematics is considered as a mother of all sciences since it is used in almost all the spheres of life. In this, we are going to see how Mathematics can be used to find the calorie consumption of any food article e.g. Medu wada, Samosa.

Here we are supposed to be known the calorie density i.e. calories per unit volume or calories per unit area, which is supposed to be obtained after lab testing. At first, we have to get the parameterization of the object. Sometimes it happens that the parameterization is done using the Cartesian co-ordinate system, sometimes it can be done using either cylindrical or spherical co-ordinate system. But always we may not be so lucky to get this by either of these ways, then we have to follow the mixed approach or altogether a different approach.
For instance Medu wada can be considered as an object obtained by revolving a circle about certain axis which we can take as Y-axis and then we can get its parameterization. Then we can do the remaining calculations, i.e. its surface area and volume using Calculus. On the other hand, a samosa can be directly parameterized using Cartesian co-ordinate system and once its parameterization is obtained we can process it in the same manner.
In short using this technique, one can control his/her diet and can live healthy life.

Tools used : MS-Powerpoint 2010, SciLab.
References : Calculus II by T. Apostol

*****************************************************************

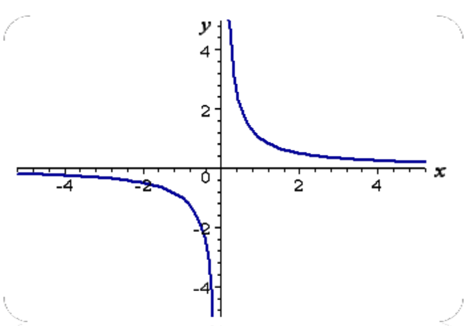
(for detailed presentations please visit http://www.vesasc.org/math_dept.php )

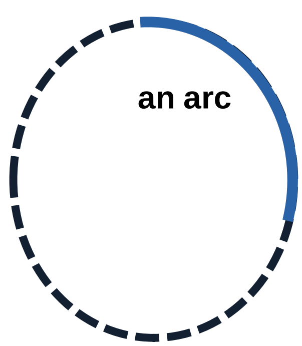
# Bezier Curve

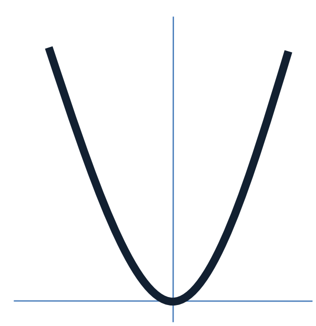
## What is a Curve

- **It is an object similar to a line but which is not required to be straight.**

- **A simple example of a curve is the parabola or an arc is a part of a curve.**





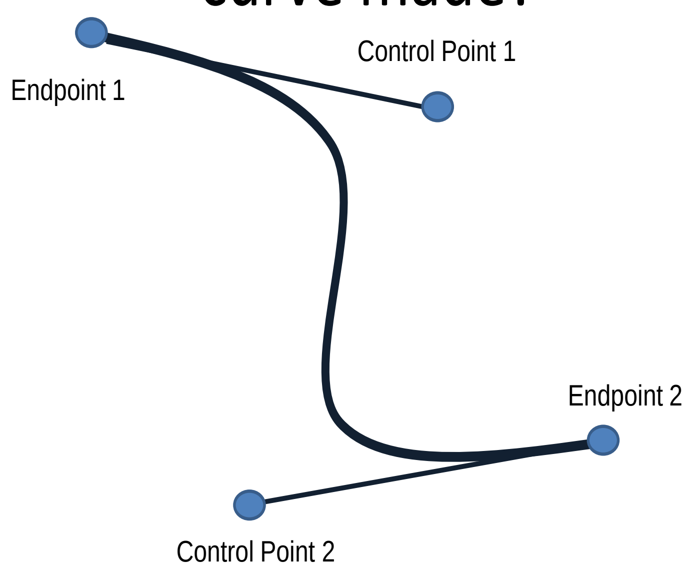


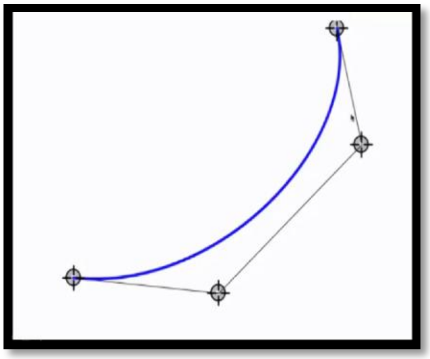
# Bezier Curve

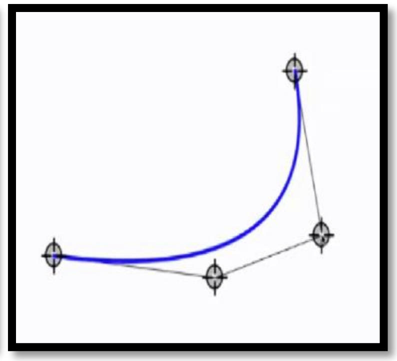
**Pierre Bézier**
**(1910 –1999)**

- **The mathematical method for drawing curves was created by Pierre Bézier in the late 1960's**
- **Manufacturer of automobiles at Renault.**

How is a Bezier curve made?

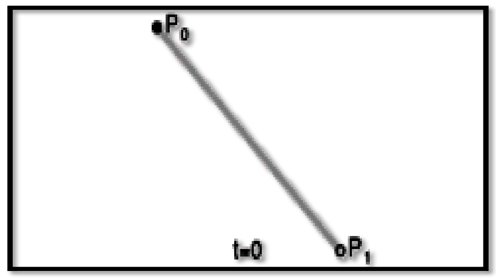


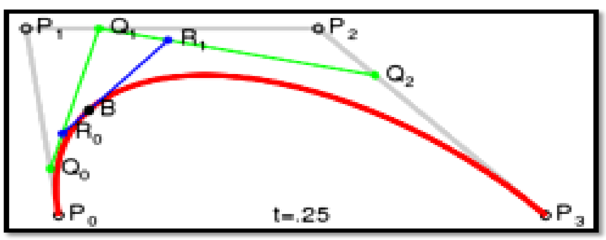


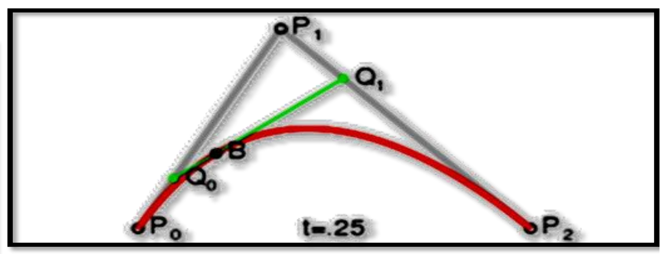


# TYPES OF BEZIER CURVES

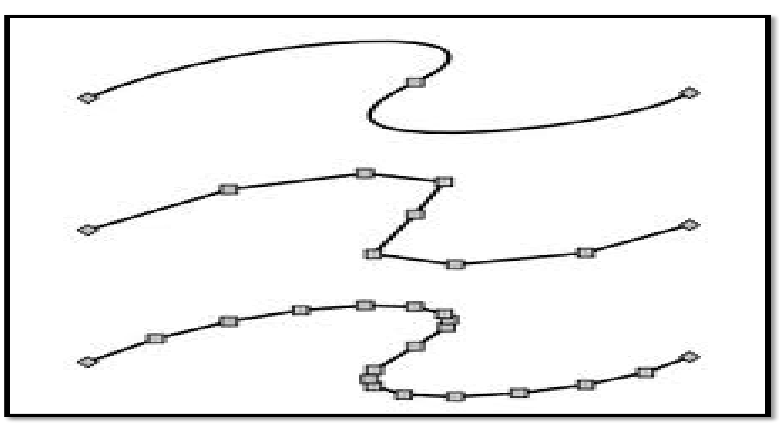
- Linear Bezier
- Quadratic Bezier
- Cubic Bezier

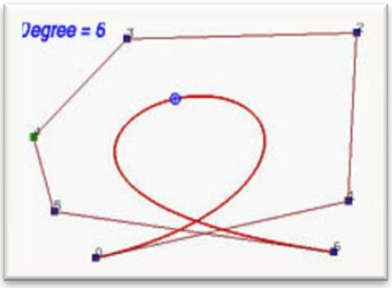



# Multiple Control Points

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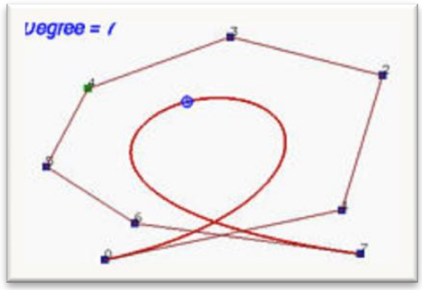
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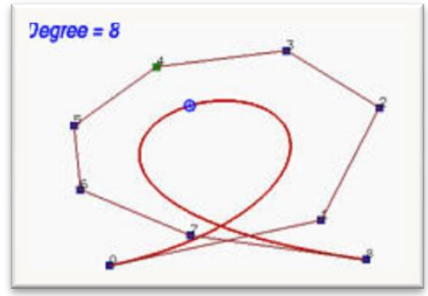
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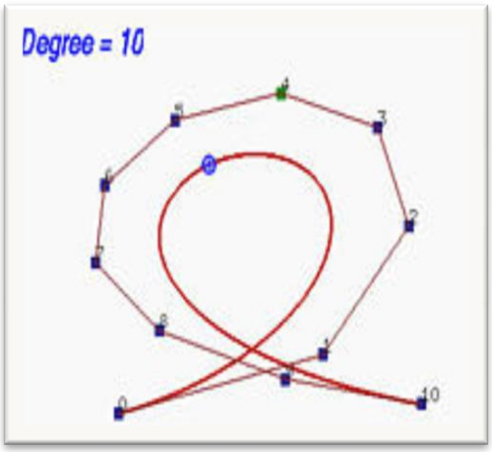
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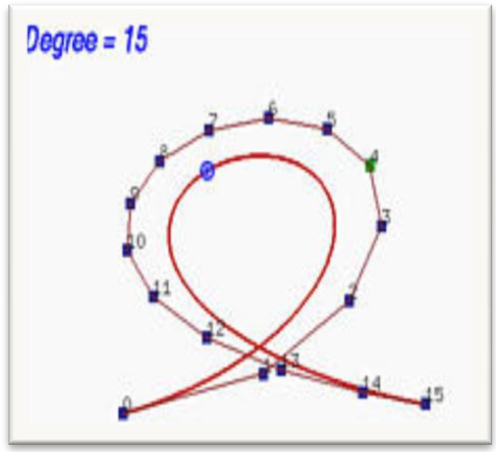
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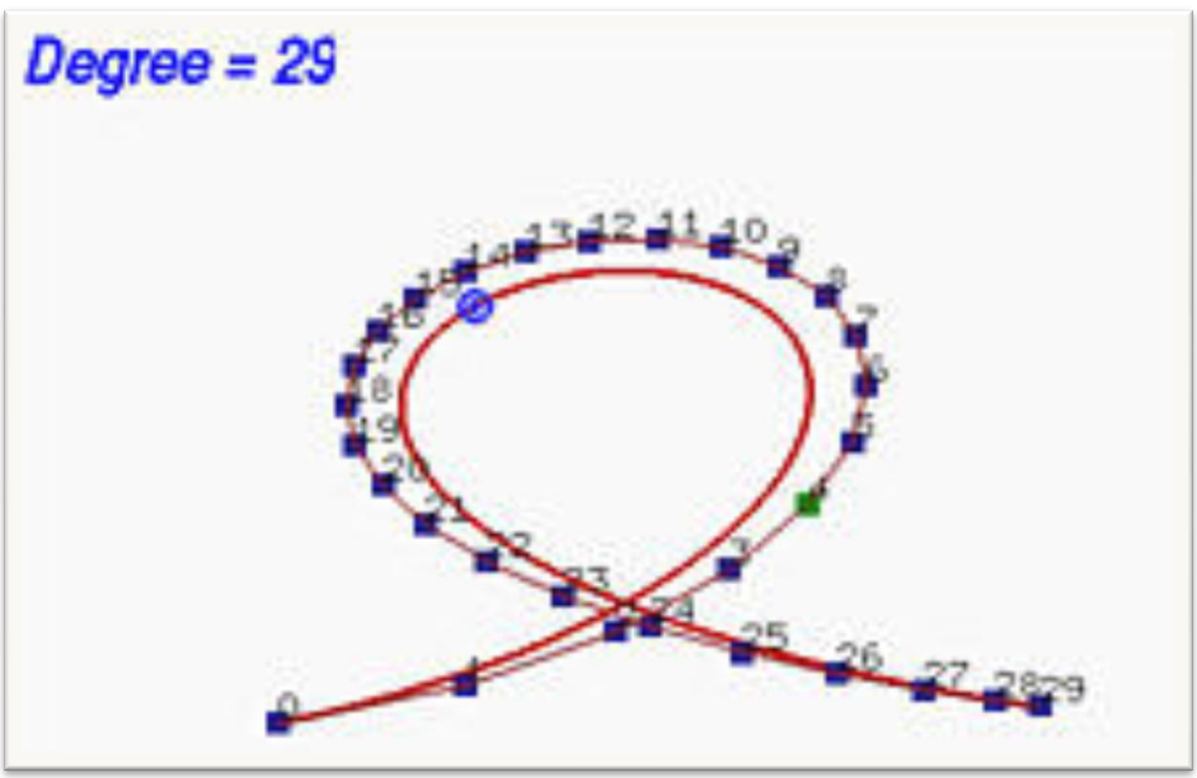

Degree = 29

# Applications

- Computer Graphics
- Animation
- Fonts
- Nature

# Computer Graphics

- It is widely used in computer graphics to model smooth curves.
- The points can be graphically displayed and used to manipulate the curve intuitively.
- Translation and rotation can be applied on the curve

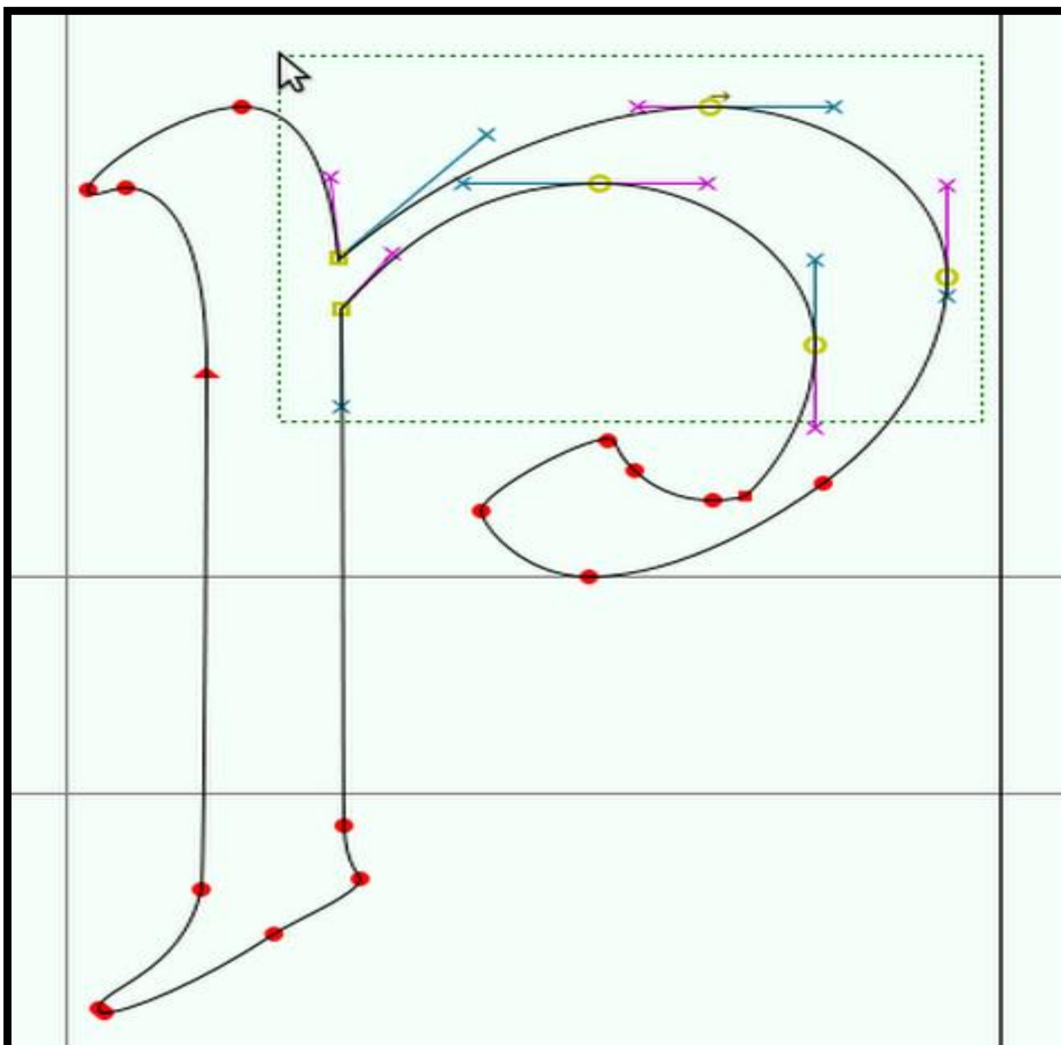
# Animation

- In animation applications, such as Adobe Flash and Synfig, Bézier curves are used to outline.
  For eg : movement.
- For 3D animation Bezier curves are often used to define 3D paths as well as 2D curves for keyframe interpolation.

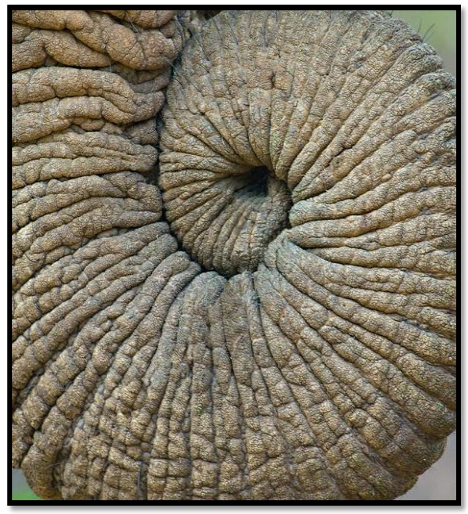
# Font

- **Fonts use Bezier splines composed of quadratic Bezier curves.**

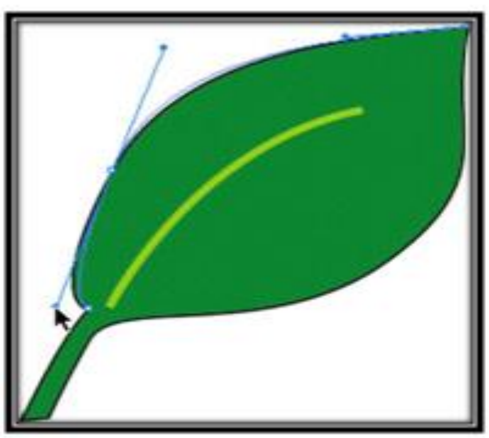
- **Modern imaging systems like PostScript, Asymptote, Metafont, and SVG use Bezier splines composed of cubic Bezier curves**



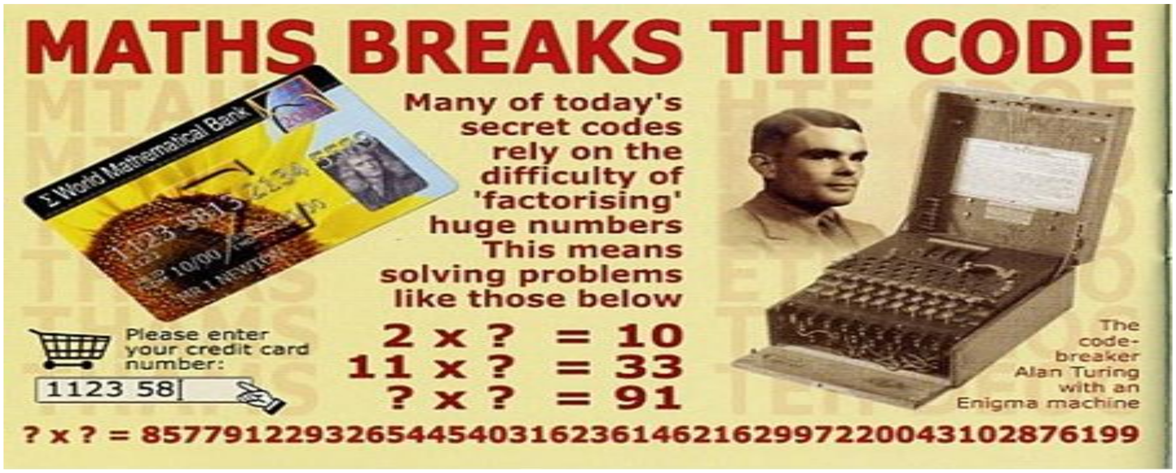
# Bezier Curve in animals



# Bezier Curve in leaves and conches



## ***CRYPTOGRAPHY***




# What is cryptography?

–*Cryptography (from the greek krypto meaning hidden and graphein meaning to write)*

– *Simply means making and breaking of secret codes.*

– *It is the science of making communications unintelligible to all except authorized parties.*

The team then verbally with the help of examples touched upon the following topics

- *Need of cryptography*
- The surprise element : Euler's mathematics
- Various terms and definitions namely :
  - *Ciphers*
  - *Plaintext*
  - *Ciphertext*
  - *Encrypting*
  - *Decrypting*

# HISTORY OF CRYPTOGRAPHY: CEASAR'S CIPHER

If we write the ciphertext equivalent underneath the plaintext letters the subsititution alphabet for the Caesar Cipher is given by

Plaintext: A B C D E F G H I J K L M N O P Q R S T U V W X Y Z

Ciphertext: D E F G H I J K L M N O P Q R S T U V W X Y Z A B C

For example, the plaintext message

CAESAR WAS GREAT

Is transformed into the ciphertext

FDHVDU ZDV JUHDW

Any plaintext is first expressed numerically

| A | B | C | D | E | F | G | H | I | J | K | L |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 00 | 01 | 02 | 03 | 04 | 05 | 06 | 07 | 08 | 09 | 10 | 11 |

| M | N | O | P | Q | R | S | T | U | V | W | X |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 |

| Y | Z |
|---|---|
| 24 | 25 |

$$C = P + 3 \pmod{26}$$

# Public key cryptography

- Cryptographic system where two keys are used: one is private and other public.
- Private key is kept secret and public key will be well known to all.
- Private key is used for decrypting
- Public key is used for encrypting
- Security depends upon the secrecy of the private key

# RSA System

- In 1977,R.Rivest,A.Shamir and L.Adleman proposed a system that uses only elementary ideas from number theory.
- Factorization of a large no is extremely difficult
- Euler's totient function is at the heart of the method.

# What is done?

- Given a plaintext(message), M, represented as a number then ciphertext is found by

$$M^k \equiv r(\text{mod } n)$$

- Likewise the decryption is found by

$$r^j \equiv M(\text{mod } n)$$

- The public key is the pair (n,k) and private key is the pair (j,n)

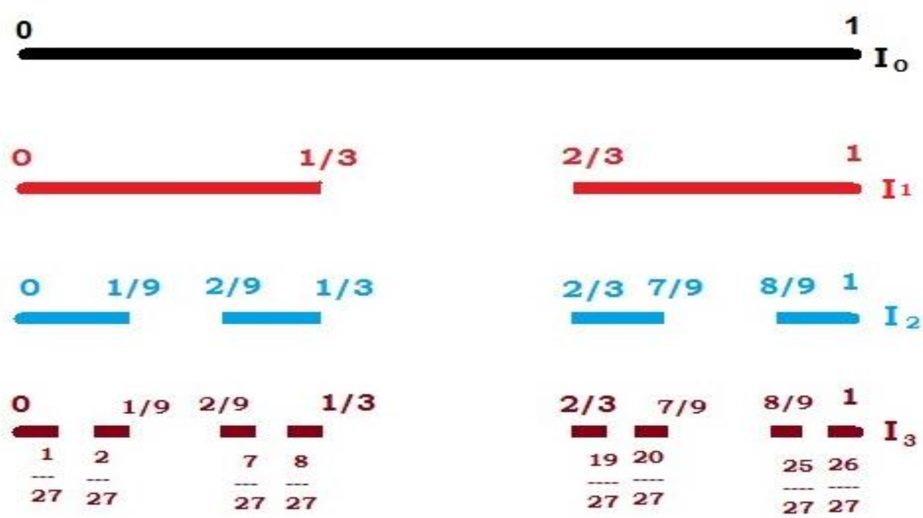
# How secure is RSA?

- Security depends upon the assumption that factorisation of composite numbers with large prime factors is not computationally feasible.
- In 1993-94 in order to factorize a 129 digit number it took 1600 computers over an 8 month period to factorize.

# Cantor's Mysterious Set

Mr. Anurag Rane
Miss Unnati Vora

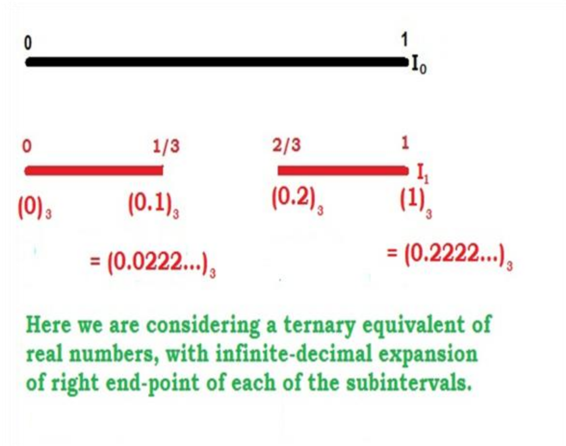
## Cantor's Set




In Mathematics, the Cantor's set is a set of points lying on a single line segment that has a number of remarkable properties

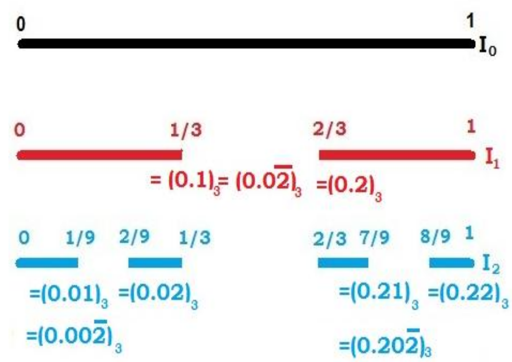
## Construction of the set

- Consider an interval, $I_0$ = [0, 1].
- Divide it into three equal parts and remove the middle $1/3^{rd}$ part.
- We get $I_{1,1} = [0, 1/3]$ & $I_{1,2} = [2/3, 1]$
  i.e. $I_1 = I_{1,1} \cup I_{1,2}$
  - $= [0, 1/3] \cup [2/3, 1]$



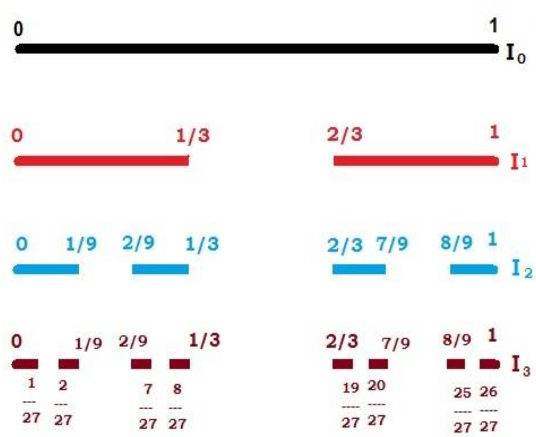

## Construction of the set

- Next divide each of these subintervals again into three equal parts and remove its middle part.
- Thus we get :
  $I_2 = I_{2,1} \cup I_{2,2} \cup I_{2,3} \cup I_{2,4}$
  $= [0, 1/9] \cup [2/9, 1/3] \cup [2/3, 7/9] \cup [8/9, 1]$



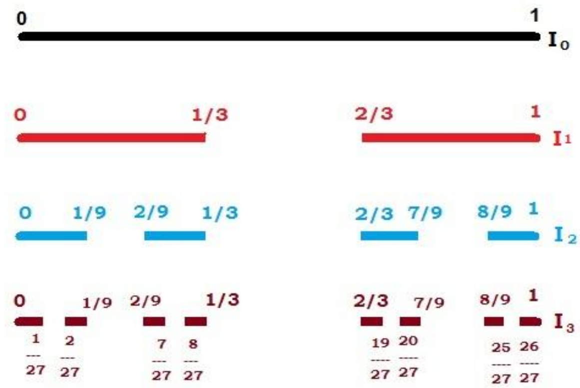

## Construction of the set

- Hence at every stage, we get, $I_n$ as a union of $2^n$ disjoint subintervals of length $(1/3)^n$ each.
- Thus length(i.e. size) of $I_n$ becomes $(2/3)^n$.
- Further as $n \to \infty$, $l(I_n) \to 0$.



# Construction of the set

- Intuitively we can say that $I_n$'s should reduce to a collection of countably many single points.
- Then we may think, which points must have been there?
- Obviously, while doing the partition, the endpoints will remain intact.




# Mysterious Cantor's set

- In fact some additional endpoints are get added. i.e. 0, 1, 1/3, 2/3, 1/9, 2/9, 7/9, 8/9, ... etc. are surely going to be the points from the Cantor's set.
- Thus we may draw the conclusion that the Cantor's Set, C has to be a subset of a set of rationals, Q, thus C must be countable.

# Mysterious Cantor's set

- But that is not the case.
- In fact we can prove that a number, ¼ even though it is not an endpoint of any of such subinterval, it is present in C.

  i.e. ¼ is an interior point of C.

## Mysterious Cantor’s set

Note that ¼ = 0.25 can be transformed into its ternary equivalent as follows :

| | | | | |
|---|---|---|---|---|
| 0.25 | 0.75 | 0.25 | 0.75 | |
| x 3 | x 3 | x 3 | x 3 | |
| -------- | ------ | ------- | ------ | And so on... |
| 0.75 | 2.25 | 0.75 | 2.25 | |

Thus $(0.25)_{10} = (0.0202\ldots)_3$

## Mysterious Cantor’s set

- Note that while constructing $I_1$ using $I_0 = [0, 1]$, we have removed (1/3, 2/3).
- Note that $(1/3)_{10} = (0.1)_3$ and $(2/3)_{10} = (0.2)_3$
- Thus $(0.25)_{10} = (0.0202\ldots)_3$ lies in $I_{1,1}$ & hence in $I_1 = [0, 1/3] \cup [2/3, 1]$.

## Mysterious Cantor’s set

- Further while constructing $I_2$ using $I_1$, we have removed (1/9, 2/9) & (7/9, 8/9).
- Note that $(1/9)_{10} = (0.01)_3$ & $(2/9)_{10} = (0.02)_3$.
- Thus $(0.25)_{10} = (0.0202\ldots)_3 > (2/9)_{10} = (0.02)_3$, it lies in $I_{2,2}$

  i.e. in $I_2 = [0, 1/9] \cup [2/9, 1/3] \cup [2/3, 7/9] \cup [8/9, 1]$

# Mysterious Cantor's set

- Further while constructing $I_3$ using $I_2$, we have removed

  (1/27, 2/27), (7/27, 8/27), (19/27, 20/27) & (25/27, 26/27).

- Note that $(7/27)_{10} = (0.021)_3$ and $(8/27)_{10} = (0.022)_3$.

- Thus $(0.25)_{10} = (0.0202\ldots)_3 < (7/27)_{10} = (0.021)_3$, implies it lies in $I_{3,3}$ & hence in $I_3$ = [0, 1/27] U [2/27, 1/9] U [2/9, 7/27] U [8/27, 1/3] U [2/3, 19/27] U [20/27, 7/27] U [8/27, 25/27] U [26/27, 1]

# Cantor's set : An uncountable set

- Consider an element $x = (0.x_1x_2x_3\ldots x_n\ldots)_2$ from a set [0, 1].

- To avoid the ambiguity, we will consider infinitesimal expansion of the real numbers from the set, [0, 1] in binary form.

  i.e. Here all $x_n$'s are either 0 or 1.

# Cantor's set :
# An uncountable set

- Now we define a map, f : [0, 1] $\rightarrow$C by f(x) = y where $x = (0.x_1x_2x_3\ldots x_n\ldots)_2$ and $y = (0.y_1y_2y_3\ldots y_n\ldots)_3$ with $y_n = 2x_n$.
- Thus for all n, $y_n$ is either 0 or 2.
- Hence all such y's are elements of Cantor's set, C
- Hence f is well-defined.

# Cantor's set :
# An uncountable set

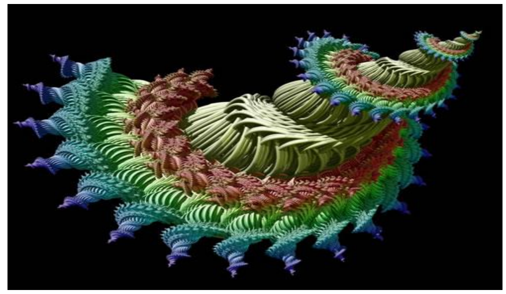
- By definition of the function, f, we can conclude that f is injective.
- Also by the nature of elements of Cantor's set(i.e. all the elements from the Cantor's set contains a string of 0's & 2's), f is clearly surjective.
- Hence f is bijective.

# Cantor's set :
# An uncountable set

- i.e. Cantor's set, C and [0, 1] are equi-numerous.
- But [0, 1] is uncountable(with its cardinality as cardinal number of continuum, c), Cantor's set C is also uncountable.
- It indicates that C contains far more number of points than the set of rationals, Q which is countable.
- Hence we call it as a mysterious set.

# Application

- Cantor's set was a motivating factor in studying '***fractals***'.
- Fractals are no-standard geometric shapes that do not perfectly fit into the world of Euclidean geometry.

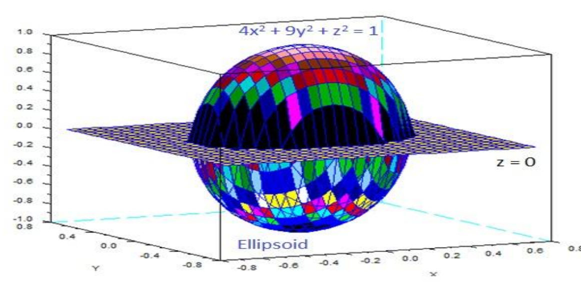


# fractals

- Fractals are often used to describe objects in nature, such as coastlines and mountains.
- A fractal by definition is a rough or fragmented geometric shape that can be split into parts, each of which is a reduced size copy of the whole.

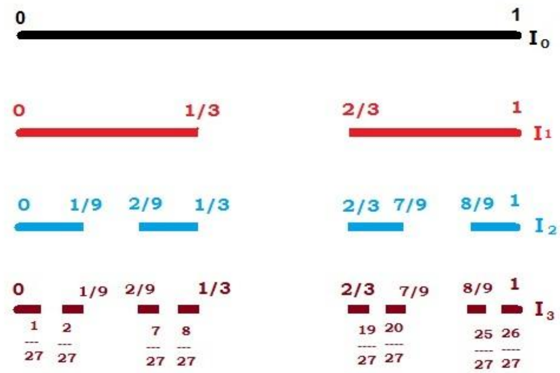
# fractals

- In traditional geometry, objects are considered to be one-dimensional, three-dimensional and so on, in integer dimensions
- E.g. a line is considered as a one-dimensional object a plane or any such smooth surface in $IR^3$ is considered as a two-dimensional object.



# fractals

- Scale-independency is the idea that by zooming in on a fractal a person cannot determine whether they are looking at the smaller or larger part of the shape.
- Cantor's set is called as base-motif fractal.




# fractals

- However, many natural objects are better described using a dimension between two natural numbers..
- So, fractals are considered to have non-integer dimensions.


# fractals

- If an object was taken with its linear size equal to 1 in Euclidean dimension, D and then reduced in its linear size by 1/r in every spatial direction for that dimension , then it takes $N = r^D$ number of self-similar objects to cover the original object.
- We get D = log(N)/log(r) as the dimension.

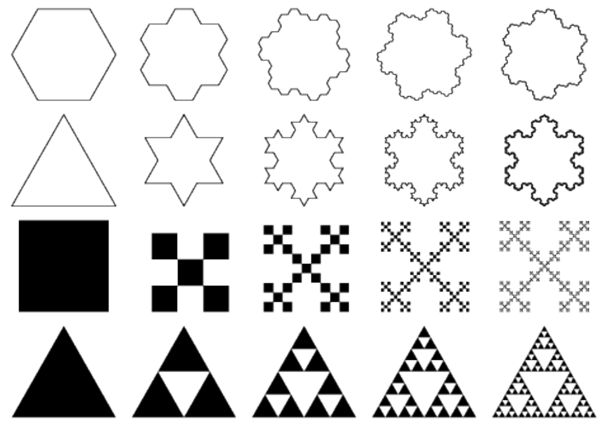
Dimension of Cantor Set

| N(s) | s |
|---|---|
| 1 | 1 |
| 2 | 1/3 |
| 4 | 1/9 |
| ... | ... |
| $2^n$ | $1/3^n$ |

Thus, $d=\ln(2^n)/\ln(3^n)$
$=\ln(2)/\ln(3)=0.6309...$ !

# fractals

- All fractals are self-similar and scale-independent.



- Self-similarity is the idea that as an object is magnified the original shape is regained.

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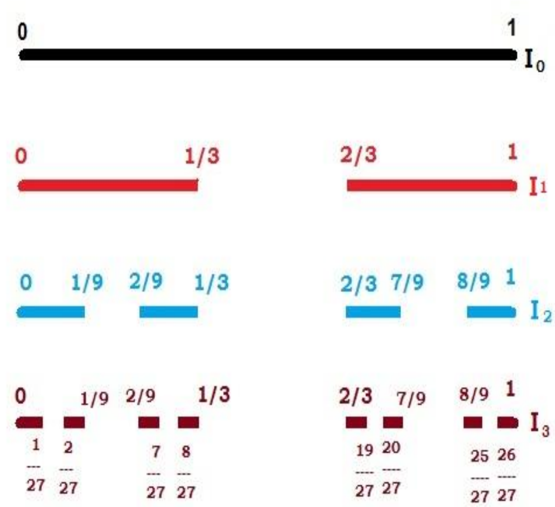
# fractals

- Scale-independency is the idea that by zooming in on a fractal a person cannot determine whether they are looking at the smaller or larger part of the shape.




- Cantor's set is called as base-motif fractal.

# fractals

- Most people encounter fractal every day without even realizing it.

- There are large number of fractals in nature such as seashells, snowflakes, mountain ranges, coastlines, lighting, clouds, trees, leaves and broccoli.

# Use of fractals

Computer science

- One of the largest uses of fractals today is in Computer Science.

- Computer graphic artists use fractal image compression to create textured landscapes.

- Fractals were used to create the geography of the moon.

# Use of fractals

Human anatomy

- The pulmonary system, which we use to breathe, is made up of tubes through which air travels into sacks called alveoli.

- The main tube, the trachea, splits into two smaller tubes that leads to different lungs, which are in turn splits into different tubes.

- Other structures include : arteries, the brain and membranes.

- Human heart beats in a fractal rhythm and doctors can detect medical problems, like heart disease, by abnormal or extreme fractal beating.

## Use of fractals

### Bacterial Growth

- Similar to trees, bacteria spread in branch patterns, which can be modeled using fractals.

# Conclusion

- Cantor Set though small is quite a surprising factor in topology

- It also has certain applications in practical life.

# AMAZING APPLICATION OF GRAPH THEORY

Sonali Chavan.
Aarifa Patel.

- Fermat’s Little Theorem.
- Cantor-Schroder-Bernstein Theorem.
- Application – The SNP Assembly Problem Using Vertex Cover Algorithm.

# FERMAT'S LITTLE THEOREM

***STATEMENT*** **:**

Let $a$ be a natural number and let $p$ be a prime such that $a$ is not divisible by $p$. Then, $a^p$ - $a$ is divisible by $p$.

***PROOF*** :

Consider the graph $G = (V, E)$, where the vertex set $V$ is the set of all sequences $(a_1, a_2, ..., a_p)$ of natural numbers between 1 and $a$ (inclusive), with $a_i \neq a_j$ for some $i \neq j$.
Total number of sequences $a^p$ and number of sequences with atleast one unequal pair is a.
Therefore number of sequences with the above property is $a^p - a$.

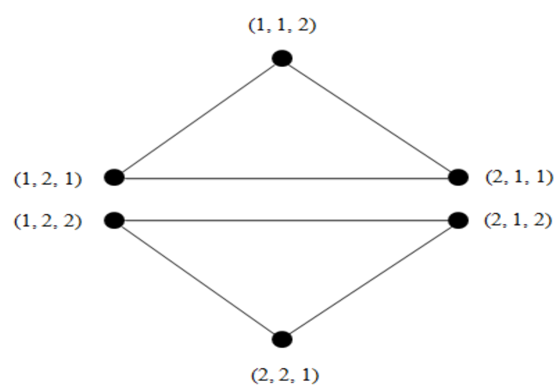
Therefore $|V| = a^p - a$.

For example : a = 2 and p = 3
(1,1,2), (1,2,1), (2,1,1), (1,2,2), (2,1,2), (2,2,1).

Edge set of G is defined as follows,

Let $u = (u_1, u_2, ..., u_p)$ and $v = (u_p, u_1, ..., u_{p\text{-}1}) \in V$ *then* $u$ *and* v *are adjacent. i.e.* $uv \in E$ if first coordinate of v and last coordinate of u match.

For example




*The graph G for* $a = 2$ *and* $p = 3$

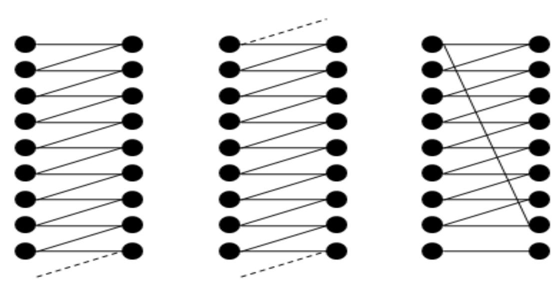
# The Cantor-Schröder-Bernstein Theorem

***STATEMENT*** :

For the sets $A$ and $B$, if there is an injective mapping $f$: $A \to B$ and an injective mapping $g$: $B \to A$, then there is a bijection from $A$ onto $B$, that is, $A$ and $B$ have the same cardinality.

*PROOF* **:** Without loss of generality, assume $A$ and $B$ to be disjoint.
Define a bipartite graph $G = (A, B, E)$, where $xy \in E$ if and only if either $f(x) = y$ or $g(y)=x$, $x \in A$, $y \in B$.
$1 \leq d(v) \leq 2$ for each $v$ of $G$. Therefore, each component of $G$ is either **a one-way infinite path** (that is, a path of the form $x_0, x_1, \ldots, x_n, \ldots$), or **a two-way infinite path** (of the form $\ldots, x_{-n}, \ldots, x_{-1}, x_0, x_1, \ldots, x_n, \ldots$), or **a cycle of even length with more than two vertices, or an edge**.
Note that a finite path of length greater or equal to two cannot be a component of $G$. Thus, in each component there is a set of edges such that each vertex in the component is incident with precisely one of these edges. Hence, in each component, the subset of vertices from $A$ is of the same cardinality as the subset of vertices from $B$.

Cantor inferred the result as a corollary of the **Well-Ordering Principle.** The above argument shows that the result can be proved without using the axiom of choice.



*Types of components of the bipartite graph G*

# The SNP Assembly Problem

In computational biochemistry there are many situations where we wish to resolve conflicts between sequences in a sample by excluding some of the sequences.

Exactly what constitutes a conflict must be precisely defined in the biochemical context.

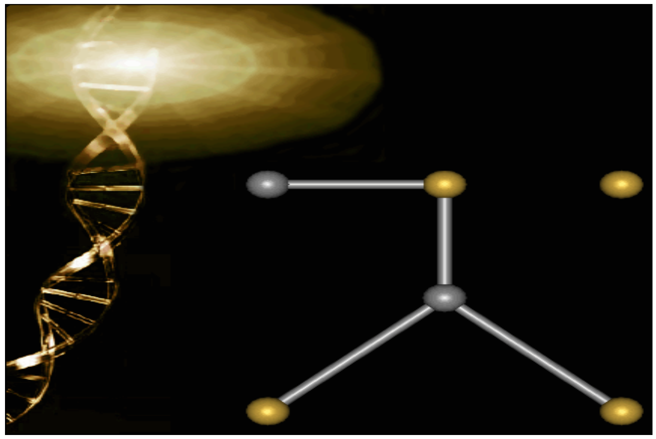
A conflict graph is a graph where the vertices represent the sequences in the sample and there is an edge between two vertices if and only if there is a conflict between the corresponding sequences.

**AIM :** To remove the fewest possible sequences that will eliminate all conflicts.

Given a simple graph *G*, a vertex cover *C* is a subset of the vertices such that every edge has at least one end in *C*. We find a minimum vertex cover in the conflict graph *G.* We look at a specific example of the SNP assembly problem given in and show how to solve this problem using the vertex cover algorithm.

A Single Nucleotide Polymorphism (SNP, pronounced "snip") is a single base mutation in DNA. It is known that SNPs are the most common source of genetic polymorphism in the human genome (about 90% of all human DNA polymorphisms).

This is known to be a **NP**-complete problem.



The DNA double helix and SNP assembly problem

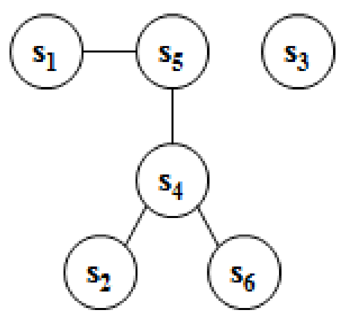
The SNP Assembly Problem is defined as follows. A SNP assembly is a triple $(S, F, R)$ where $S = \{s_1, \ldots, s_n\}$ is a set of $n$ SNPs, $F = \{f_1, \ldots, f_m\}$ is a set of $m$ fragments and $R$ is a relation $R: S \times F \rightarrow \{0, A, B\}$ indicating whether a SNP $s_i \in S$ does not occur on a fragment $f_j \in F$ (marked by 0) or if occurring, the non-zero value of $s_i$ ($A$ or $B$). Two SNPs $s_i$ and $s_j$ are defined to be in conflict when there exist two fragments $f_k$ and $f_l$ such that exactly three of $R(s_i, f_k)$, $R(s_i, f_l)$, $R(s_j, f_k)$, $R(s_j, f_l)$ have the same non-zero value and exactly one has the opposing non-zero value. The problem is to remove the fewest possible SNPs that will eliminate all conflicts. The following example from is shown in the table below. Note that the relation $R$ is only defined for a subset of $S \times F$ obtained from experimental values.

| R | $f_1$ | $f_2$ | $f_3$ | $f_4$ | $f_5$ |
|---|---|---|---|---|---|
| $s_1$ | A | B | - | - | B |
| $s_2$ | B | A | A | A | 0 |
| $s_3$ | 0 | 0 | B | B | A |
| $s_4$ | A | 0 | A | 0 | B |
| $s_5$ | A | B | B | B | A |
| $s_6$ | B | - | A | A | 0 |

*Table for the relation R*

Note, for instance, that $s_1$ and $s_5$ are in conflict because $R(s_1, f_2) = B$, $R(s_1, f_5) = B$, $R(s_5, f_2) = B$, $R(s_5, f_5) = A$. Again, $s_4$ and $s_6$ are in conflict because $R(s_4, f_1) = A$, $R(s_4, f_3) = A$, $R(s_6, f_1) = B$, $R(s_6, f_3) = A$.

Similarly, all pairs of conflicting SNPs are easily determined from the table. The conflict graph *G* corresponding to this SNP assembly problem is shown below.



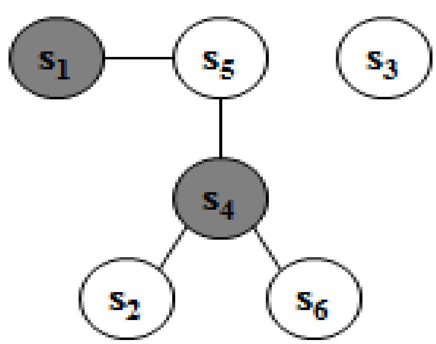

*The conflict graph G*

We now use the vertex cover algorithm to find minimal vertex covers in the conflict graph *G*. The input is the number of vertices 6, followed by the adjacency matrix of *G* shown below. The entry in row *i* and column *j* of the adjacency matrix is 1 if the vertices $s_i$ and $s_j$ have an edge in the conflict graph and 0 otherwise.

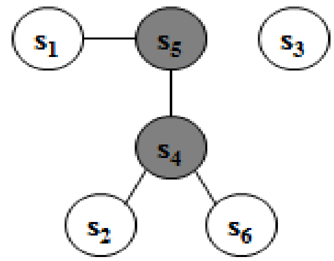
| | | | | | |
|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 1 | 0 |
| 0 | 0 | 0 | 1 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 1 | 0 | 0 | 1 | 1 |
| 1 | 0 | 0 | 1 | 0 | 0 |
| 0 | 0 | 0 | 1 | 0 | 0 |

*The input for the vertex cover algorithm*

The vertex cover program finds two distinct minimum vertex covers, shown as below.




*Minimum Vertex Cover:* $s_1$, $s_4$




*Minimum Vertex Cover:* $s_4$, $s_5$

*The output of the vertex cover algorithm*

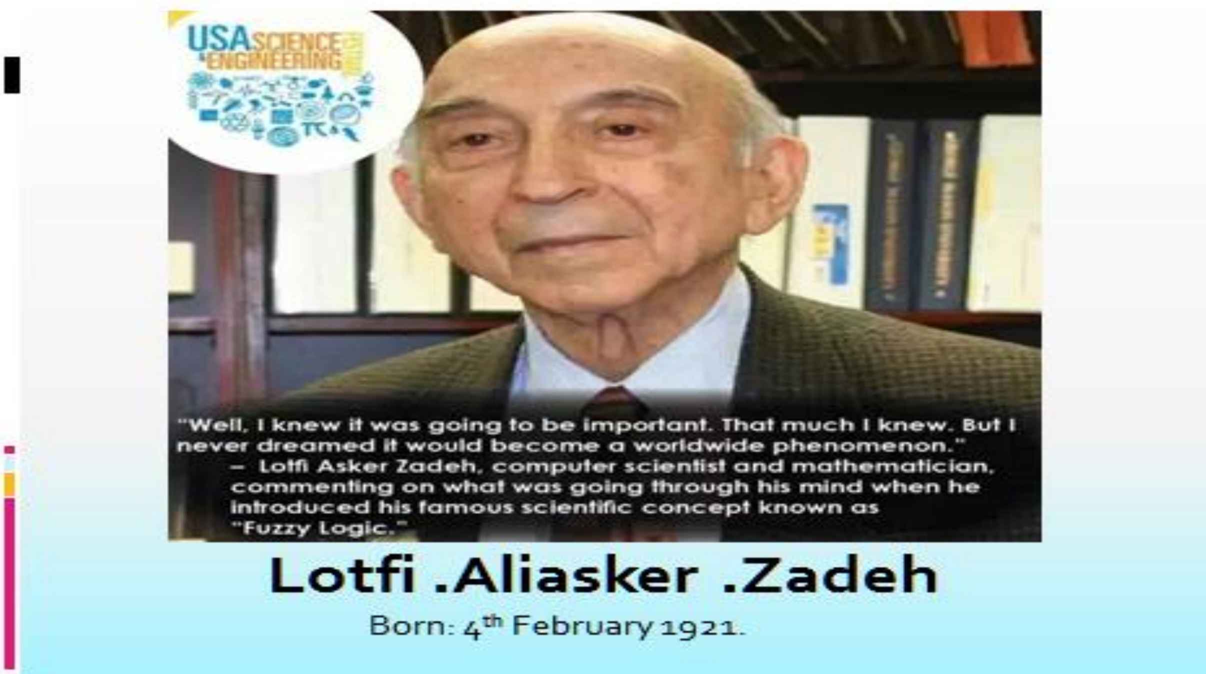
Thus, either removing $s_1$, $s_4$ or removing $s_4$, $s_5$ solves the given SNP assembly problem.

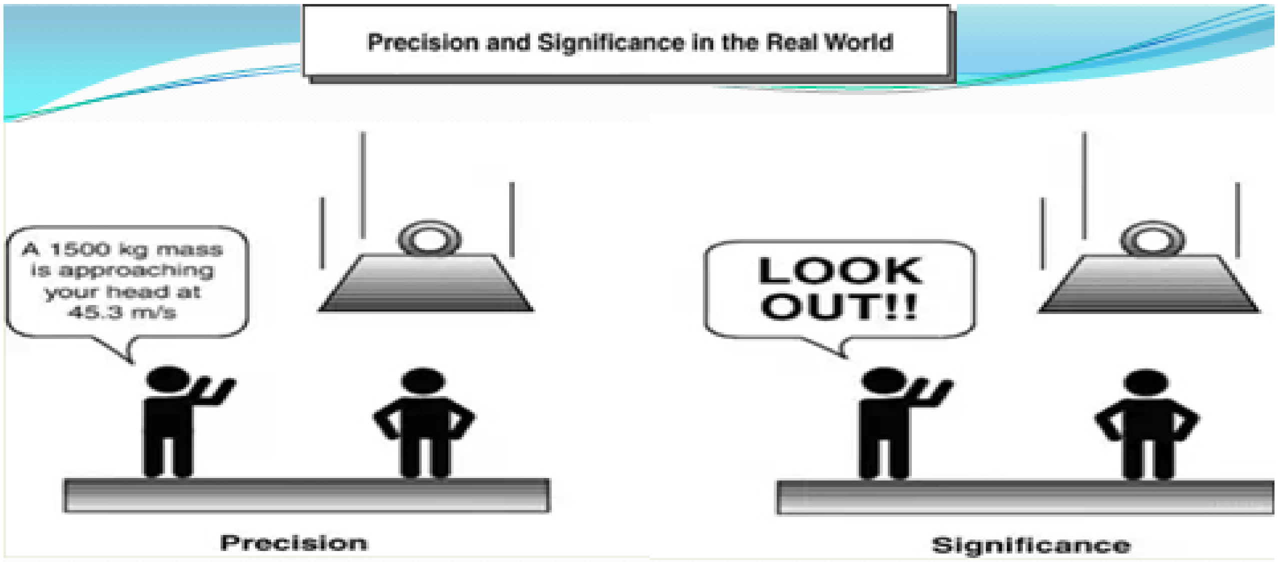
# References


•Graph Theory
by D. B. West.

•Applications of Graph Theory
by Ashay Dharwardkar.


Fuzzy Algebra
UNCERTAINTY AHEAD
-Vivek Sharma
Indubati
Kushwaha


"Well, I knew it was going to be important. That much I knew. But I never dreamed it would become a worldwide phenomenon."
– Lotfi Asker Zadeh, computer scientist and mathematician, commenting on what was going through his mind when he introduced his famous scientific concept known as "Fuzzy Logic."
Lotfi .Aliasker .Zadeh
Born: 4th February 1921.

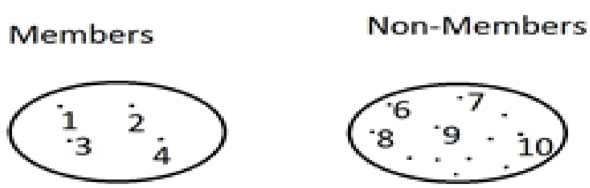



**Some Situations don't need precision, a level of Uncertainty do the needful!!!**

# WHAT IS A FUZZY SET?

Definition:-A fuzzy subset $A$ of a set $X$ is a function $A:X\rightarrow L$, where $L$ is the interval [0,1]. This function is also called a membership function.

Eg.

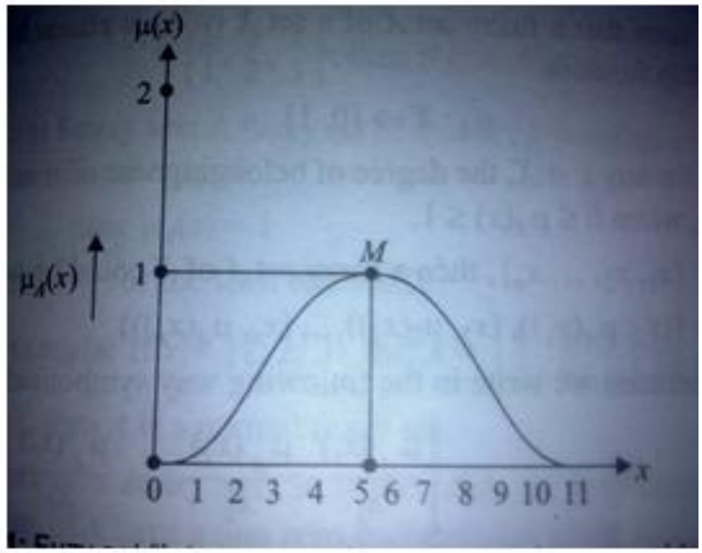



X={1,2,3,4} X'={6,7,8,9,10,....}

# Find Natural nos. which are MORE or LESS than 5.

It is NOT like our Normal Sets. In Math, We have been using a Variable Which are either a Member of a Set or a Non-Member.......... What say about this question???

Here $\mu_A(x)$ is a membership function, Which shows "How much a variable is in the set?"




## Difference Between Probability and Fuzzy Logic

- <u>Probability</u>:- 'How probable do I think that a variable is in the set or not?'

  It is a "Probability Measure".
- <u>Fuzzy Logic</u>:- 'How much a variable is in the set?'

  It is a "Possibility Measure".

## Example of "YOUTHNESS"

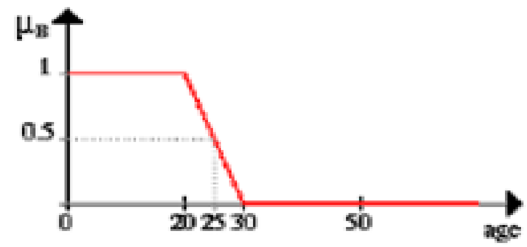
Upto what age are Boys YOUNG???

A. Young(x)={1,if age<=20};

B. Young(x)={30-(age)/10,if 20<age<=30};

C. Young(x)={0,if age(x)>30}.

X axis shows the age of Boys while Y axis shows the Membership function which tells how much young a Boys is???




## Applications:-

- The First Known Application after L.A.Zadeh published his paper on Fuzzy Logic was by JAPANESE in their High-Speed Train in Sendai.

JAPAN'S HIGH SPEED E6 SERIES TRAIN:

Built: Aerodynamic Design.

Normal Speed: 200km/h.

Max. Speed: 325km/h.

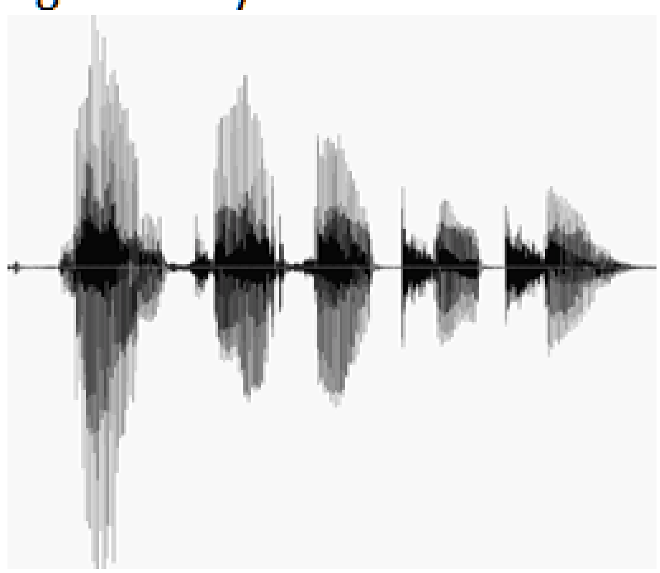
Functionality: Fuzzy System developed by JAPANESE ENGINEERS.

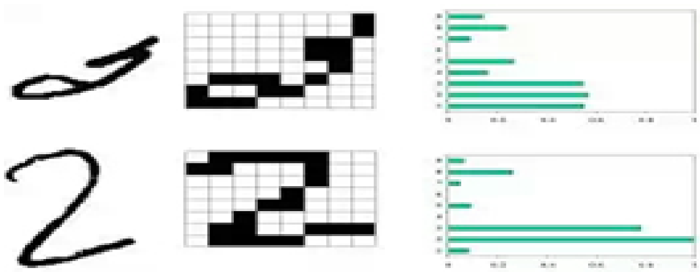
## Fuzziness in a Train........

- Electro hydraulic System detects a curve taken by the train at such high speeds and minimizes the effect on passengers.
- Fuzzy Intelligent braking System calculates the speed it should maintain before taking a halt, which should be a must in such High-Speed Trains.

- Binary State won't help in developing Voice or Handwriting Recognition System.

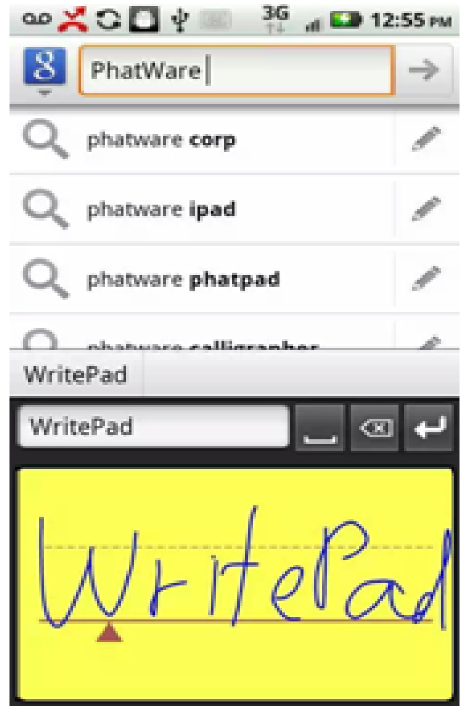
FUZZY SYSTEM notices the change in sound, pauses, breaks and many other factors to ENHANCE the SPEECH RECOGNIZING capability of a DEVICE.

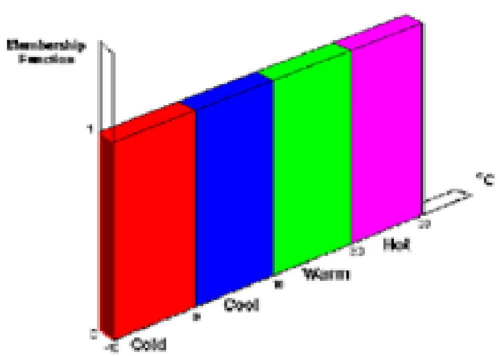




## Handwriting Recognition:

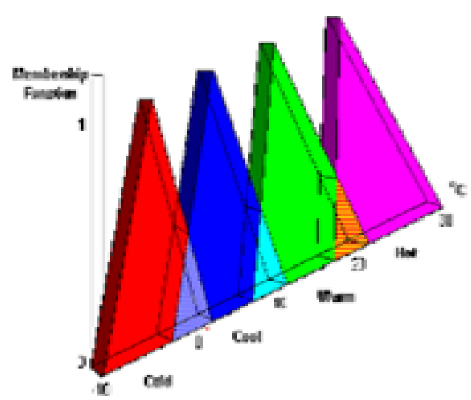
New Systems uses this logic to ENHANCE the Writing Capability in a Digital System. By taking an account of degree of curves, level of pressure, flow of writing the machines are made artificially intelligent.




Bivalent Set to Characterize the Temperature of the room.

Fuzzy Set to Characterize the Temperature of the room.



## Guess The Surprise Element???

Hints :

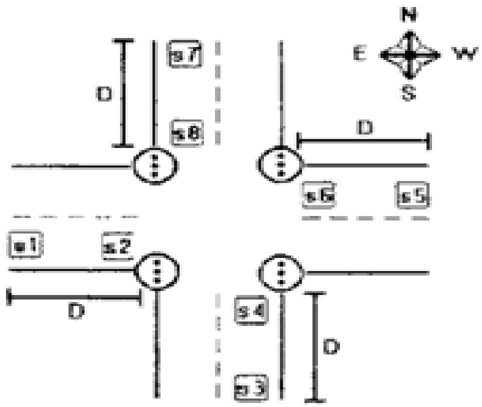
- An Idea which can reduce POLLUTION PROBLEM in our country…
- An Idea which can solve all Traffic Issues…
- An Idea which can save our time…
- An Idea which is efficient enough to replace present System…

FUZZY (Traffic Control System)

## Problem Redefined……

- Conventional Traffic System has a Constant Cycle Time, Which is NOT an optimal solution to reduce TRAFFIC.
- More cars should be passed through signals if there are FEW cars waiting behind red lights, which in turn clear the traffic.
- A mathematical model for this decision is very difficult to find.

## Fuzzy Intelligent Traffic System(F.I.T.S)




- Each side consist of TWO Incremental sensors which count the no. of cars waiting behind the red light.
- Here, our main Aim is to make Signals ARTIFICIALY INTELIGENT by varying the Cycle Time of the Signals.
- Varying the Cycle Time will Re-prioritize the time allotted to each Direction based on traffic in it.

## How the model Works???

### Step 1: FUZZIFICATION

- The Input of the model consist of

a) Cycle time.
b) Cars behind RED light.
c) Cars behind GREEN light

| STATE | CYCLE TIME | CARS BEHIND RED LIGHT |
|---|---|---|
| ZERO | 0 | 1 |
| LOW | 0 | 7 |
| MEDIUM | 4 | 11 |
| HIGH | 7 | 18 |
| CHAOS | 14 | 20 |

| STATE | CYCLE TIME | CARS BEHIND GREEN LIGHT |
|---|---|---|
| VERY SHORT | 0 | 14 |
| SHORT | 0 | 34 |
| MEDIUM | 14 | 60 |
| LONG | 33 | 88 |
| VERY LONG | 65 | 100 |
| LIMIT | 85 | 100 |

## Step 2: Rule Evaluation

- Formulated using a series of if…then statements and Logical operators.
- There are 150 rules for this model.(5*5*6)
- Eg:-If cycle time is medium AND cars behind Red is low AND cars behind Green is medium, Change is Probably NOT.

| LEVEL OF OUTPUT | Singleton values |
|---|---|
| NO | 0 |
| PROBABLY NO | 0.25 |
| MAY BE | 0.5 |
| PROBABLY YES | 0.75 |
| YES | 1 |

| MEMBERSHIP DEGREE | VALUES |
|---|---|
| YES | 0 |
| PROBABLY YES | 0.6 |
| MAYBE | 0.9 |
| PROBABLY NO | 0.3 |
| NO | 0.1 |

## Step 3: Defuzzification

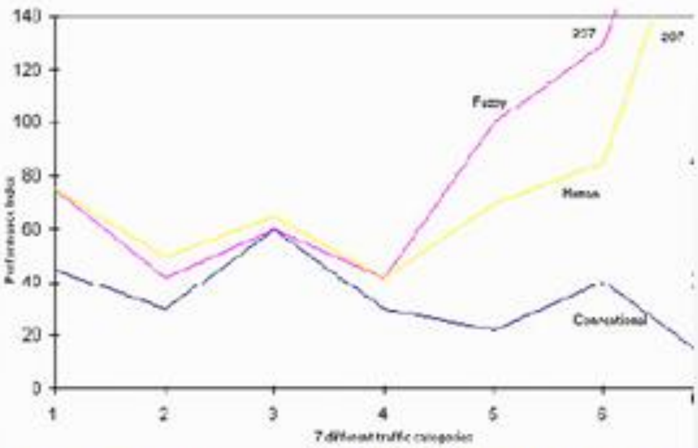
- The process to convert Fuzzy Set output to Real Crisp value is called as "DEFUZZIFICATION".
- FORMULA for Defuzzification:

Crisp set=[Sum(Membership Degree*Singleton position)]/(Membership degree).

For the Rule Considered:

Output=(0.1*0.0)+(0.3*0.25)+(0.9*0.5)+(0.6*0.75)+(0*1.00)/(0.1+0.3+0.9+0.6+0)=0.51

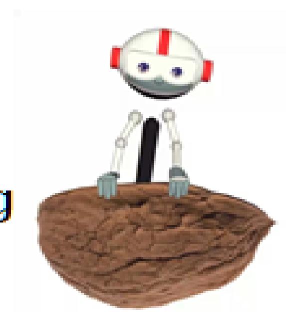
# Conclusion




| Traffic System | PERFORMANCE INDEX |
| --- | --- |
| FUZZY | 237 |
| HUMAN | 207 |
| CONVENTIONAL | 138 |

✓Fuzzy Traffic Control System's performance index was 72% higher than the conventional System.
✓It's performance index was 36% higher in comparison to Human Expert.

# In a NUTSHELL



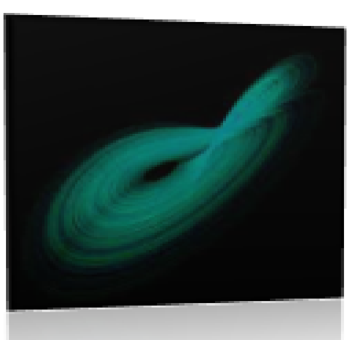
- In fuzzy logic, exact reasoning viewed as a limiting case of approximate reasoning.
- In fuzzy logic everything is a matter of degree.
- Any logical system can be fuzzified
- Fuzzy logic can be used to make smarter machines with Human Like Capabilities and take ARTIFICIAL INTELLIGENCE to next level.

## References:

- www.en.wikipedia.org
- Discrete Mathematics with Graph Theory.
  -S.K.Yadav
- Our all time favorite
  "Google Search Engine!"
- Fuzzy logic and it's application in real world.
  -Pooja Dhiman, Gurpreet Singh
- The Birth and Evolution of Fuzzy Logic.
  -L . Zadeh
- Maths Dept. of My College.

**Have you ever wondered why weather forecasters have such a hard time when they try to predict beyond a short period of time ?**

# The Lorenz Equation and Its Applications



-BY

-Arjun Singh Gusain

-Dhanashree R H

# THE LORENZ EQUATION

- In 1963-mathematician & meteorologist- Edward Lorenz took up this problem , focusing specifically on his model for weather prediction.

# THE LORENZ EQUATIONS

Prandti number

$$dx/dt = \sigma(x - y)$$

$$dy/dt = rx - y - xz$$

$$dz/dt = -bz + xy$$

Rayleigh number

## SIMPLE PROPERTIES OF THE LORENZ EQUATIONS

- Nonlinearity - the two nonlinearities are xy and xz
- Symmetry - Equations are invariant under (x,y)    (-x,-y). Hence if (x(t),y(t),z(t)) is a solution, so is (-x(t),-y(t),z(t))

# VISUALIZING SOLUTIONS

•Equilibrium solutions : The most important solutions in a nonlinear system are the equilibrium solutions.

$$(x - y) = 0 \quad ...(1)$$
$$rx - y - xz = 0 \quad ...(2)$$
$$-bz + xy = 0 \quad ...(3)$$

From the first equation, we see that any equilibrium solution must have $y = x$.

Eliminating y from the second and third equations, we find

$$x(r - 1 - z) = 0 \quad ...(2)$$
$$-bz + x^2 = 0 \quad ...(3)$$

from (2), Either $x = 0$ or $z = r - 1$.

If $x = 0$, then from (3) ,$z = 0$.

So one equilibrium solution is $(0, 0, 0)$.

If $z = r - 1$, then from (3),

$x = \pm(b(r - 1))^{1/2}$.

So, if $r > 1$, we also have the equilibria

$(\pm(b(r - 1))^{1/2}, \pm(b(r - 1))^{1/2}, r - 1)$

# NUMERICAL APPROXIMATION

Lorenz used EULER'S method to numerically approximate solutions for his system of equations.


Euler's method

$$y_{n+1} = y_n + hf(x_n, y_n)$$

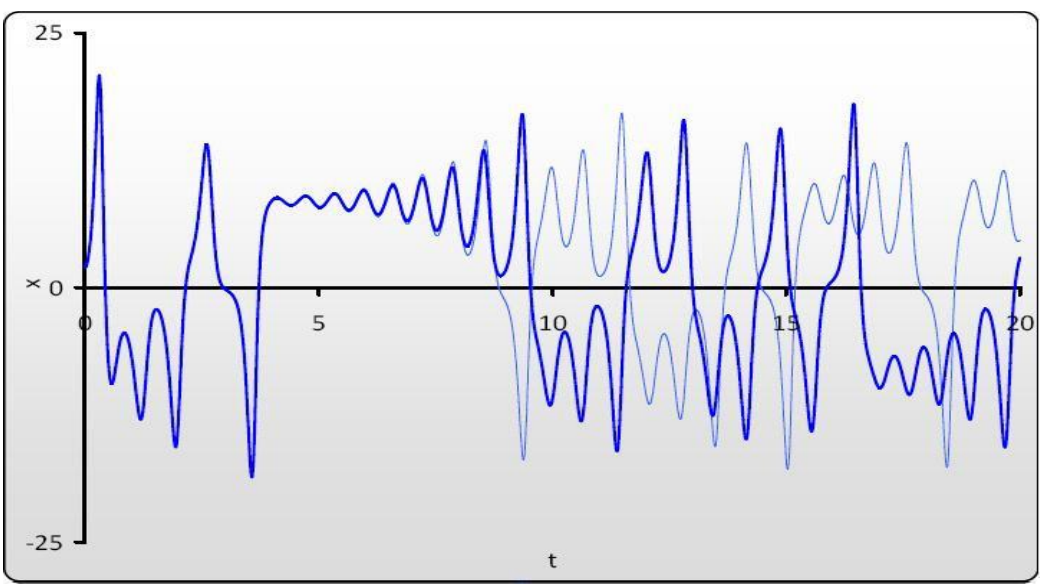
$$y_{n+1} = y_n + hf(x_n, y_n) + \frac{h^2}{2!} y''(x_n)$$

- Program butterfly.java uses Euler method's to numerically solve Lorenz's equation & plots the trajectory (x,y,z).
- RUNGE-KUTTA METHOD is the other method for numerical approximation.

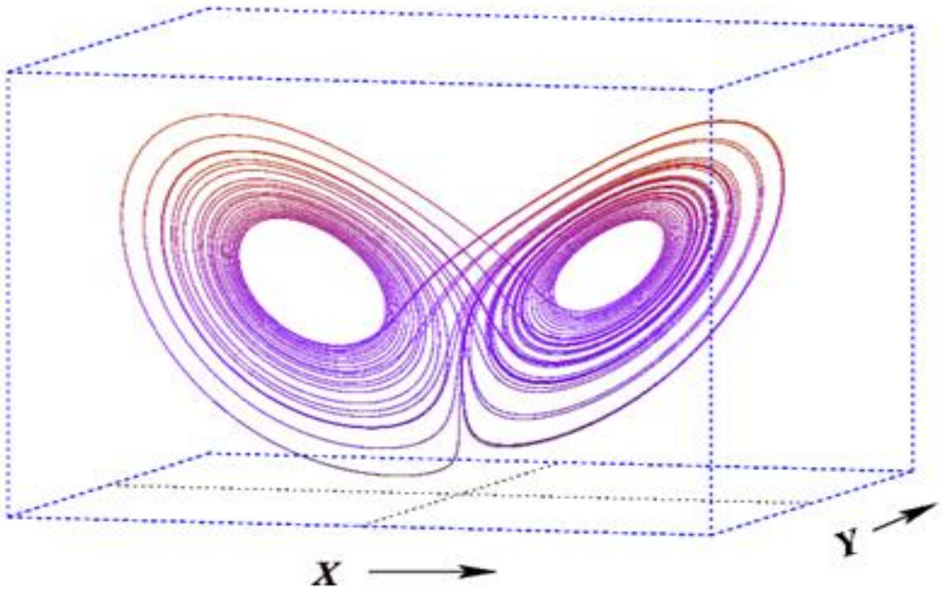
# CHAOS !!

- When we investigate the system for different values of r, we find :
  - r < 1, that most solutions tend asymptotically to the equilibrium point (0, 0, 0).
  - For intermediate values of r , most solutions tend toward one of the non trivial equilibria
  - For a larger value of r, r = 28. We would like to determine what happens to the solution in the long run for given initial conditions.

Initial conditions:
(2,2,2) & r=28
(2.01,2,2) & r=28

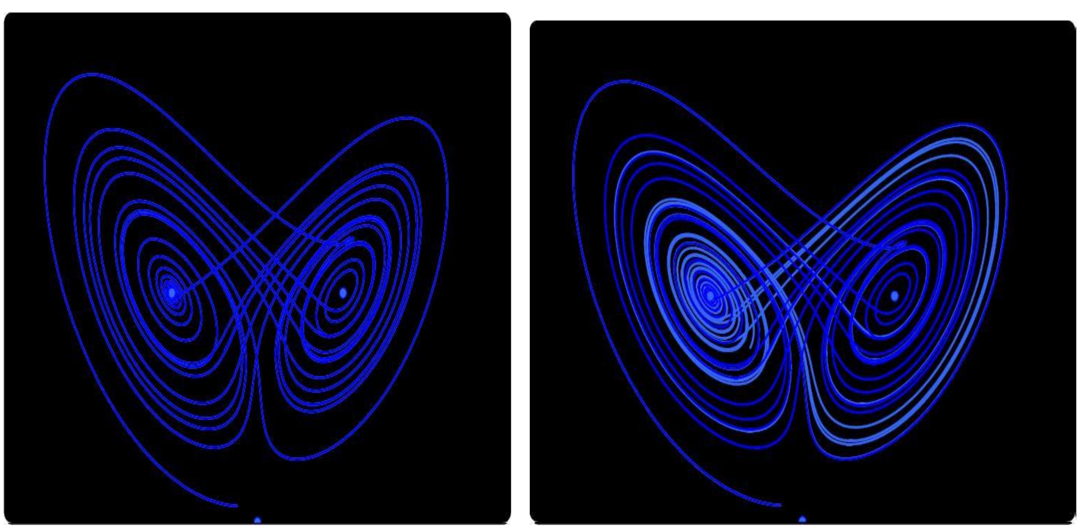


# GRAPH OF X VS Y VS Z – THE LORENZ ATTRACTOR




# GRAPH OF X VS Y VS Z – THE LORENZ ATTRACTOR

**(2,2,2) & r=28** **(2.01,2,2) & r=28**



# WHY IT DIVERGES??

**In numerical studies of lorenz attractor, one finds that**

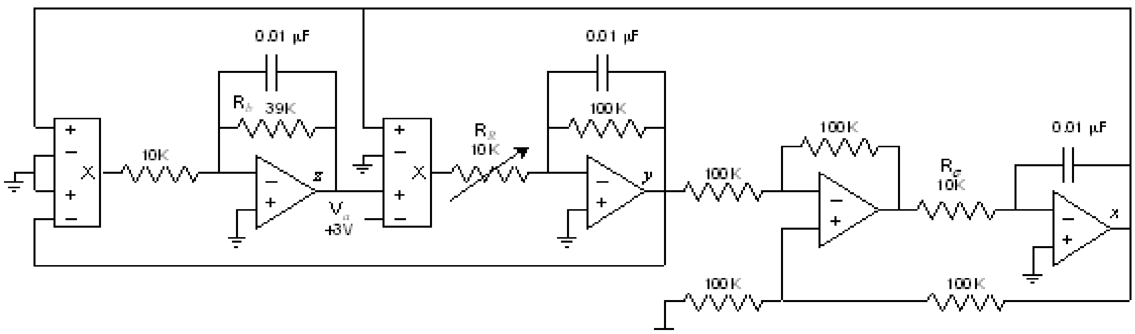
**$\delta(t) \sim \delta_0 e^{\lambda t}$ , where $\lambda \approx 0.9$**

Surprise element!!!!

- The motion on the attractor exhibits ***sensitive dependence on initial conditions.***
- Two trajectories starting very close together will rapidly diverge from each other & there after have totally different future.

# APPLICATIONS

- Lorenz accidentally found the chaotic behavior of his simple looking system
- It kicked of the chaos theory & from then it has never looked back!!
- THE CHAOTIC OSCILLATOR: Lorenz system is used to explore chaos. To obtain practical circuit design we have to scale the lorenz equation.




$x = X/\sqrt{\alpha R}$ , $y = Y/\sqrt{\alpha R}$, $z = Z/\sqrt{\alpha R}$

$\alpha$: scale parameter

$\alpha = 1/3$ & x,y,z in volts

## CHAOS *COMMUNICATION*

- A chaotic oscillator is essentially flexible & therefore allows for very sensitive selection between a really wide range of possible behaviors.

- We force a chaotic oscillator to carry information, small parametric feedback control sensitively alters a chaotic trajectory into some other symbolic sequence corresponding a desired message.

The heart of chaos communication is to manipulate the dynamics so to transmit an information bearing signal.
The field of controlling chaos was popularized by "OGY" Algorithm published in 1990 by ott,urebogi and yorke.

## FEW OTHER APPLICATIONS

- Quantum control - cybernetical physics
- Femto chemistry
- Femto technologies using cybernetics

# Words regarding weather forecasting

"When our results concerning the instability of non-periodic flow are applied to the atmosphere, which is ostensibly non-periodic, they indicate that prediction of the sufficiently distant future is impossible by any method, **unless the present conditions are known exactly.** In view of the inevitable inaccuracy and incompleteness of weather observations, precise very-long-range forecasting would seem to be non-existent."

— Edward N. Lorenz

## REFERENCES:


http://maxwell.ucsc.edu
International journal of bifurcation and chaos
WSPC
http://www.usna.edu

# FOOTBALL, NETWORKS AND MATHS

**Nidhi Makhijani**
**Rahul Makhijani**

## INTRODUCTION

- In making a mathematical model for a real world problem we make certain assumptions to render the mathematics tractable .
- For years, the Poisson distribution has been used to describe many event models. The Poisson distribution is called the law of small numbers because Poisson events occur rarely even though there are many opportunities for these evens to occur.
- In this paper, Poisson distribution has been used to fit to goals scored in a soccer match and network traffic.

# Poisson Process

- Poisson distribution is used to model the number of events happening per time interval, such as the number of customers arriving to a store per hour, or the number of visits per minute to an internet site.

- A random variable (r.v.) $N$ that takes values 0, 1, 2, ... has Poisson distribution if

$$P\ (N(t) - 0 = k) = e^{-\lambda t} \frac{(\lambda t)^k}{k!}.$$

- The mean and the variance of Poisson distribution is $1/\lambda$

# Exponential distribution

- Exponential distribution, as its name implies, describes a process whose probability is exponentially distributed.
- The probability distribution of exponential process is $f(x) = \lambda e^{\lambda x}$ for $x \geq 0$.
- The cumulative distribution is $F(x) = 1 - e^{\lambda x}$
- The mean and variance of an exponentially distributed R.V. $X$ are $1/\lambda$ and $1/\lambda^2$ respectively.

## PROPERTIES OF POISSON PROCESSES AND EXPONENTIAL DISTRIBUTION

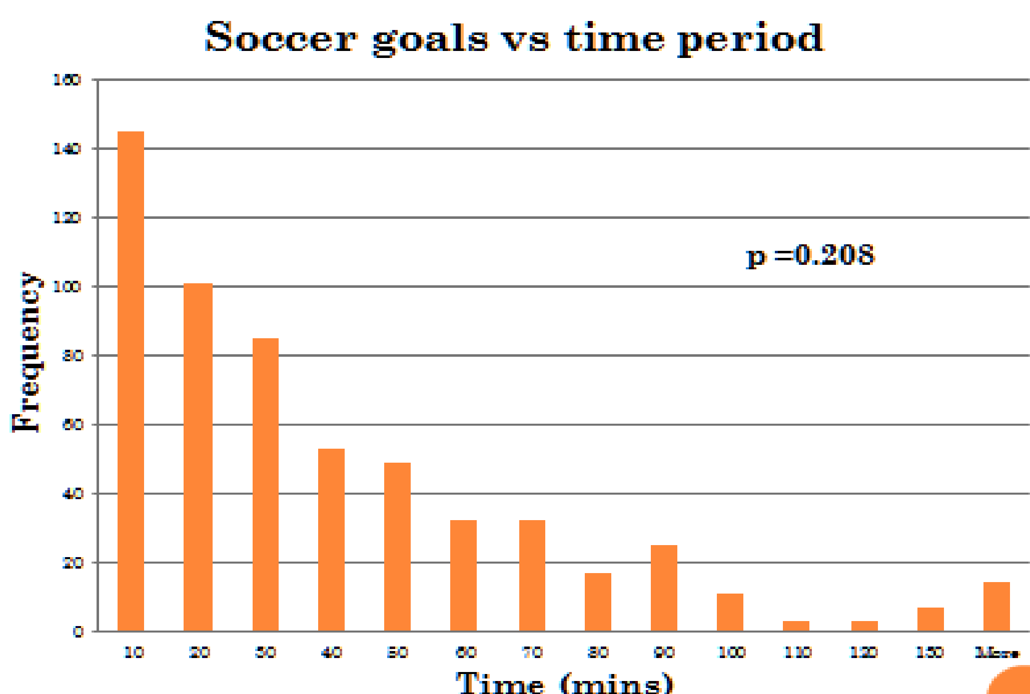
- $T_n$ denotes the time between the arrival between $(n-1)^{th}$ event and $n^{th}$ event.
- The sequence of inter arrival processes are independent exponential random variables having mean $1/\lambda$.
- An exponential process satisfies the memory less property

i.e. $P\{X > s + t \mid X > t\} = P\{X > s\}$ for all $s, t \geq 0$

## RELATION BETWEEN POISSON PROCESS AND SOCCER WORLD CUP

- We analyze the goal occurrences of four World Cup tournaments (1990 – 2002). Specifically, we will look at the goal occurrences in the 232 games played in the 1990 – 2002 tournaments. This set of data is obtained from Fifa's World Cup website. We will focus only on the goals scored in the 90 minutes regulation time.

- Based on the number of goals scored (577) and the games played (232), we can estimate the average number of goals scored per 90 minute game,
- $\lambda = 577/232 = 2.49$
- The theoretical expected time between goals is $1/\lambda = 36.14$ min
- The theoretical variance between the goals is $\frac{1}{\lambda^2} = 1306.43\ min$




**Exponential Distribution**

| Time Interval | Frequency | Emperical Probability | Theoretical probability | Theoretically Expected Goals |
|---|---|---|---|---|
| 0-10 | 145 | 0.25 | 0.24 | 139.46 |
| 10-20 | 101 | 0.18 | 0.18 | 105.76 |
| 20-30 | 85 | 0.15 | 0.14 | 80.20 |
| 30-40 | 53 | 0.09 | 0.11 | 60.81 |
| 40-50 | 49 | 0.08 | 0.08 | 46.11 |
| 50-60 | 32 | 0.06 | 0.06 | 34.97 |
| 60-70 | 32 | 0.06 | 0.05 | 26.52 |
| 70-80 | 17 | 0.03 | 0.03 | 20.11 |
| 80-90 | 25 | 0.04 | 0.03 | 15.25 |
| 90-100 | 11 | 0.02 | 0.02 | 11.56 |
| 100-110 | 3 | 0.01 | 0.02 | 8.77 |
| 110-120 | 3 | 0.01 | 0.01 | 6.65 |
| 120-130 | 7 | 0.01 | 0.01 | 5.04 |
| More | 14 | 0.02 | 0.75 | 432.00 |

## TABLE TO CHECK MEMORYLESS PROPERTY

| 0.972631 | p(T>10) | | |
|---|---|---|---|
| 0.730931 | p(T>20) | 0.928493 | P(T>20)/P(T>10) |
| 0.547631 | P(T>30) | 0.948548 | P(T>30)/P(T>20) |
| 0.408631 | P(T>40) | 0.962431 | P(T>40)/P(T>30) |
| 0.303241 | P(T>50) | 0.972315 | P(T>50)/P(T>40) |
| 0.223323 | P(T>60) | 0.979438 | P(T>60)/P(T>50) |
| 0.162721 | P(T>70) | 0.984647 | P(T>70)/P(T>60) |
| 0.116767 | P(T>80) | 0.988492 | P(T>80)/P(T>70) |
| 0.081919 | P(T>90) | 0.991349 | P(T>90)/P(T>80) |
| 0.055494 | P(T>100) | 0.993483 | P(T>100)/P(T>90) |
| 0.035455 | P(T>110) | 0.995082 | P(T>110)/P(T>100) |
| 0.02026 | P(T>120) | 0.996285 | P(T>120)/P(T>110) |

## Important Observations

- We test whether the inter-arrival times are independent within each time interval, as well as between the first lag of the 10 minute subintervals. For this, we used the autocorrelation function.

- The correlation between the inter arrival times {T} and {T+10} was found to be -0.005 indicating independence of the inter arrival time.

- The process was found to be memory less as
  $P(T>20|T>10) \cong P\ (T>10)$

## Internet Network and Traffic Models

Internet traffic can be modeled as a sequence of arrivals of discrete entities, such as packets, cells, etc. Mathematically, this leads to the usage of two equivalent representations: *counting processes* and *inter arrival time processes*.

A counting process $\{N(t)\}_{t=0..\infty}$ is a continuous-time, integer-valued stochastic process, where N(t) expresses the number of arrivals in the time interval *(0,t]*.

An interarrival time process is a non-negative random sequence $\{A_n\}$, where $A_n=T_n-T_{n-1}$ indicates the length of the interval separating arrivals *n-1* and *n*.

$$\{N_t = n\} = \{T_n \le t < T_{n+1}\}$$

## LIMITATIONS OF POISSON MODELS

- Traffic Burstiness –

a sequence of arrival times will be bursty if the $T_n$ tend to form clusters, that is, we sees a mix of relatively long and short interarrival times.

- Mathematically speaking, it implies that interarrival times are autocorrelated.

- However, there is not a single widely accepted notion of burstiness; instead, several different measures are used, some of which ignore the effect of second order properties of the traffic..

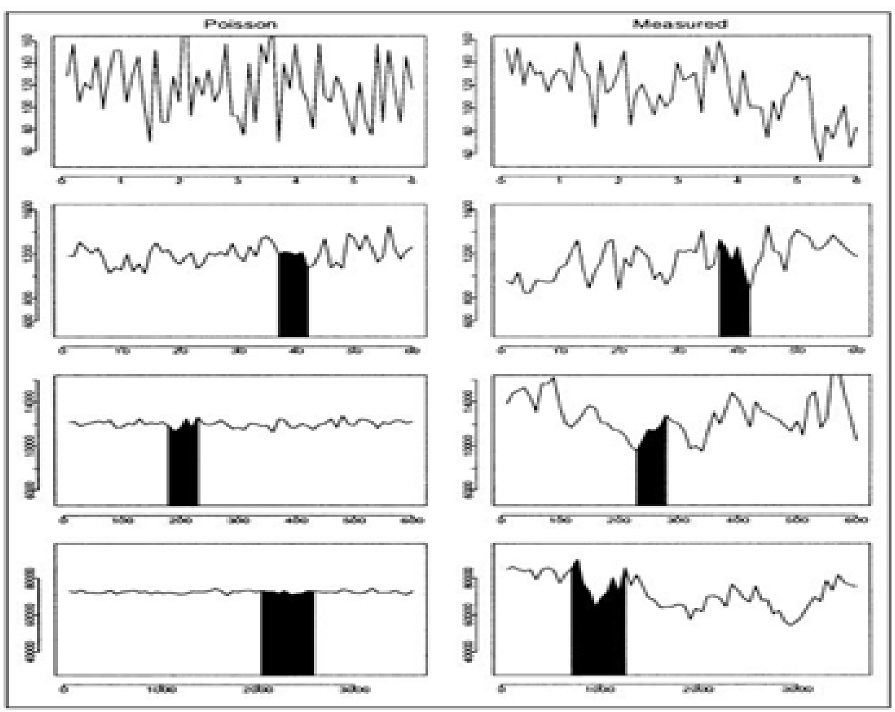



Figure 1, taken from [willinger98where], illustrates the failure of the Poisson model in capturing internet traffic burstiness.

The plots on the right hand side represent the trace of traffic arrivals registered in 1995 on a network link connecting a large corporation to the Internet.
From Ref. (5)

## OBSERVATIONS

- Poisson-like traffic would be easier to control: above a certain time scale, knowing the long-term arrival rate is enough to characterize the traffic. There would be no need for big buffers in routers or for complicated mechanisms to guarantee quality of service. On the contrary, the kind of burstiness present in the traffic represented on the right hand side does not suggest any conservative operating point

- Therefore, sophisticated mechanisms for traffic engineering are required. One way to represent burstiness through a Poisson model is to use the so called *compound* Poisson process, where packet arrivals happen in bursts (or batches), the interbatch times are independent and exponentially distributed (that is, they represent a Poisson process), and the batch sizes are random. This scenario can be modeled using two processes $\{A_n\}$ and $\{B_n\}$, the first one representing the batch interarrival times and the second the batch sizes.

## CONCLUSION

- We found that the football goals can be modeled well with a Poisson process.

- Many processes such as road traffic or internet traffic cannot be ideally modelled with Poisson processes.

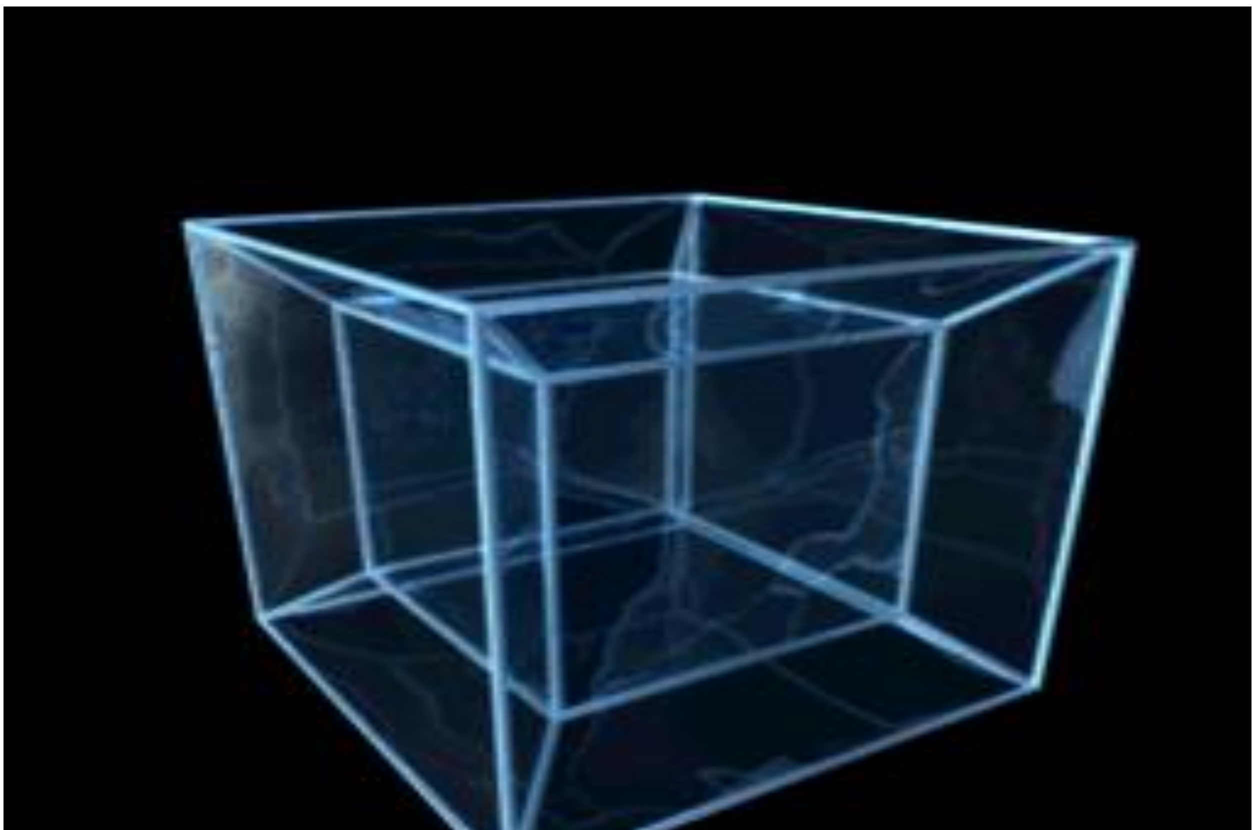
- We cannot make too many assumptions or else our conclusions obtained by mathematical models will not be applicable to the real-world situation.

## REFERENCES


- Introduction to Probability Models by Sheldon Ross.
- http://www.cse.wustl.edu/~jain/cse567-06/ftp/traffic_models1/index.html
- http://www2.ensc.sfu.ca/people/grad/pwangf/IPSW_report.pdf
- http://www.icir.org/vern/papers/poisson.TON.pdf
- W. Willinger, V. Paxson, and M.S. Taqqu, "Self-similarity and Heavy Tails: Structural Modeling of Network Traffic". In A Practical Guide to Heavy Tails: Statistical Techniques and Applications, Adler, R., Feldman, R., and Taqqu, M.S., editors, Birkhauser, 1998.

# *Hyperdimension*

## 4- dimensional hypercube



# A STRANGE PHENOMENON 1

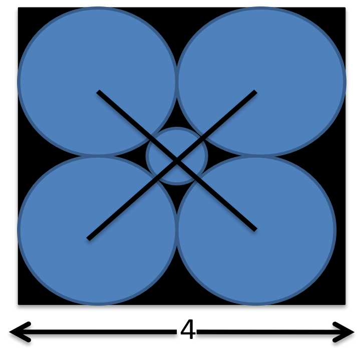
Consider a 4*4 square inside of which is embedded four touching circles, each circle of radius 1

Then,

Radius of inner small circle

Is $r = \sqrt{1^2 + 1^2} - 1$

$= \sqrt{2} - 1$



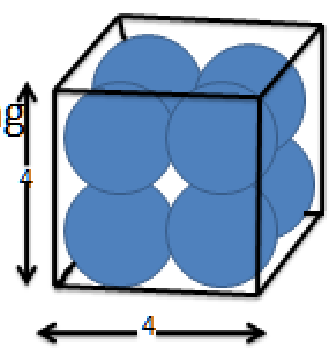

Consider equivalent situation in 3 dimension .
Inside a cube of side 4 are embedded 8 touching spheres, each of radius 1

Then,

Radius of inner small inner touching sphere is,

$$r = \sqrt{1^2+1^2+1^2} - 1$$

$$= \sqrt{3} - 1$$



NOW,

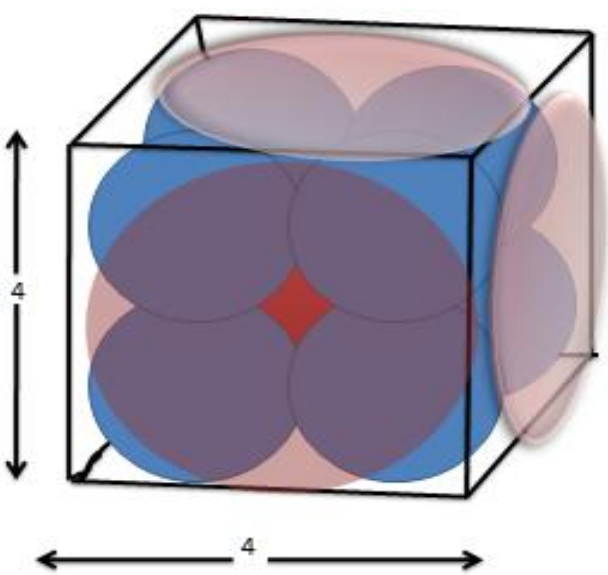
❖An ‘n’ dimensional hypercube of side L [with one vertex at origin ]is the set of all n tuples $\{X1, X2, X3 \ldots\ldots\ldots\ldots\ldots Xn\}$ where $Xr \in \{0, L\}$ It will have $2^n$ vertices

❖ also , a hyper sphere is set of all n tuples **such that** $\mathbf{x1^2} + x2^2 + x3^2 + \ldots\ldots Xn^2 \leq R^2$

❖For L=4 , using Pythagoras theorem

$r = \sqrt{1^2+1^2+1^2 \ldots\ldots\ldots +1^2} - 1$

$= \sqrt{n} - 1$



❖ the distance from the centre of hypercube to any of its side is exactly 2

❖ (for L= 4 ; R= 1)

so if n =9 then,

$r = \sqrt{9} - 1 = 2$

❖Means it touches the sides of the hypercube and when n > 9 it protrudes outside the hypercube.

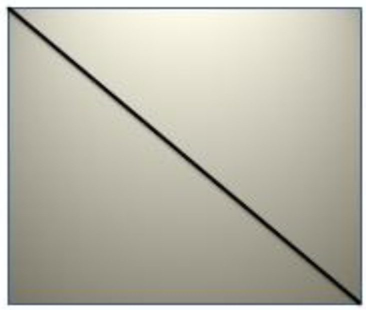
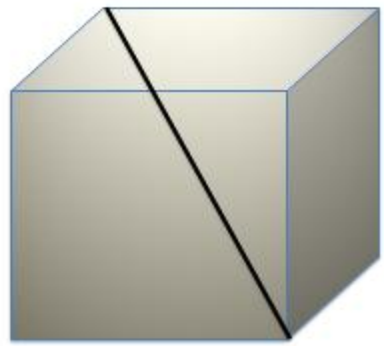
**STRANGE PHENOMENON 2:-**

if we want to place a stick of length L into hypercube diagonally we require $L= m\sqrt{n}$

which means that –

**" AS THE DIMENSION INCRESES , THE LENGTH OF THE HYPERCUBE NEEDED TO CONTAIN THE STICK DIMINISES.**

Example ; a hypercube of side 1 meter of dimension 100 could contain a stick of length 10 meters .

| Dimension | SIDE OF THE CUBE/HYPERCUBE | LENGTH OF THE STICK |
|---|---|---|
| n = 1 | 1 | 1 |
| n=2 | 1 | 1.414 |
| n=3 | 1 | 1.73 |
| n=4 | 1 | 2 |
| .......... | 1 | .......... |
| .......... | 1 | .......... |
| 100 | | 10 |

# VOLUME IN DISCRETE HYPERDIMENSION

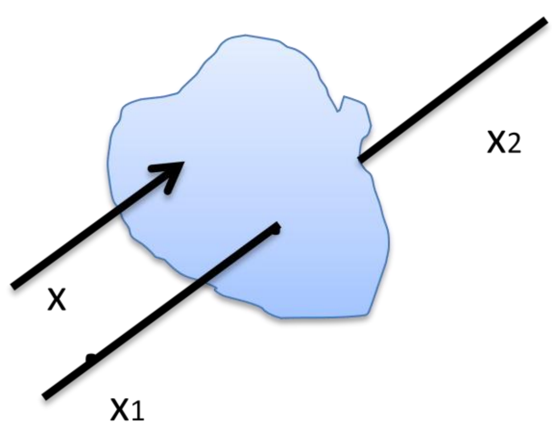
❖ To find out the volume of a hyper sphere with radius X having dimension 'n'

❖ **Formula** :-

$$V_n(X) = X^n \, V_n(1)$$

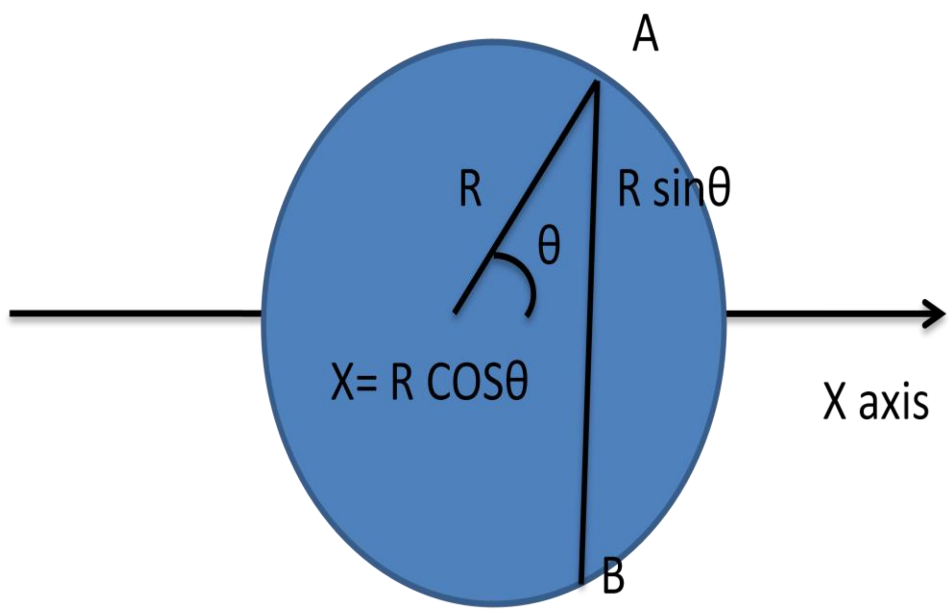
where, $V_n(1)$ is the volume of hyper sphere with radius 1 & dimension "n".

❖ to find the volume of solid we take an arbitrary axis which runs through it and add the area of section A(X) through the solid perpendicular to this axis as demonstrated in figure .



❖ For a sphere ,

we consider a section of sphere in 3- dimension




Circular section A(X) at a distance of $x= R \cos\theta$ from centre .

Diameter is AB ; radius of cross section is $R \sin\theta$ .

Then We have, $V_3(R) = \int_{x_1}^{x_2} A(X)\, dx$

where, $A(X) = V2\,(R \sin\theta) = (R \sin\theta)^2\, v2\,(1)$

{using formula $V_n(X) = X^n\, V_n(1)$

NOW,

To Integrate,

$$V_3(R) = \int_{X_1}^{X_2} A(X)\, dx$$

$$V_3(R) = \int_{-R}^{R} (R \sin\theta)^2 \; V_2(1)$$

now, $X = R \cos\theta$ ; $dx / d\theta = -R \sin\theta$

substituting $\theta$ for X in the integral

$$V_3(R) = \int_{\pi}^{0} (R \sin\theta)^2 \; V_2(1) * -R \sin\theta\, d\theta$$

$$= R^3 \; V_2(1) \int_0^{\pi} \sin^3\theta \; d\theta$$

$$= R^3 V_2(1) \int_0^{\pi} \sin\theta . \sin^2\theta \; d\theta$$

$$= R^3 V_2(1) \int_0^{\pi} \sin\theta\,(1 - \cos^2\theta)\; d\theta$$

$$= R^3 \; V_2(1) \int_0^{\pi} \sin\theta - \sin\theta . \cos^2\theta \; d\theta$$

$$= R^3 \; V_2(1) \left[ -\cos\theta + 1/3 \cos^3\theta \right]_0^{\pi}$$

$$V_3(R) = R^3 V_2(1) \; [\, 2/3 + 2/3 \,] \; = \; 4/3 \, R^3 V_2(1)$$

now, $V_2(1) = \pi * 1^2 = \pi$

hence , $R^3 V_2(1) = 4/3 \; R^3 * \pi$

SO, we found out volume of sphere i.e a 3 dimensional figure using volume of circle i.e a 2 dimensional figure.

Now, we will calculate the volume of hyper sphere with radius R & dimensions "n". using n-1 dimensional hyper sphere .

$$V_n (R) = \int_{X1}^{X2} V_{n-1} (X)\, dx$$

$$= \int_{-R}^{R} V_{n-1} (X)\, dx$$

$$= \int_{\Pi}^{0} V_{n-1} (X)\, dx$$

$$= \int_{\Pi}^{0} V_{n-1} (R \operatorname{Sin}\theta) * -R \operatorname{Sin}\theta \, d\theta$$

$$= \int_{\Pi}^{0} (R \sin\theta)^{n-1} V_{n-1} (1) * -R \sin\theta$$

$$\{ \text{ using formula } V_n(X) = X^n V_n(1) \}$$

$$= V_{n-1} (1) R^n \int_{0}^{\Pi} \sin^n\theta \, d\theta$$

$$V_n(R) = R^n V_n(1) = V_{n-1} (1) R^n \int_{0}^{\Pi} \sin^n\theta \, d\theta$$

We conclude that , for R=1

$$V_n(1) = V_{n-1}(1) \int_0^{\pi} \sin^n\theta \, d\theta$$

$$= V_{n-1}(1) \; I_n$$

we will solve $I_n$ first,

integrating by parts

$$I_n = \int_0^{\pi} \sin^n\theta \, d\theta = \int_0^{\pi} \sin\theta * \sin^{n-1}\theta \, d\theta$$

$$= [- \cos\theta * \sin^{n-1}\theta]_0^{\pi} + (n-1) \int_0^{\pi} \sin\theta * \sin^{n-1}\theta \, d\theta$$

$$= (n-1) \int_0^{\pi} (1-\sin^2\theta) * \sin^{n-2}\theta \, d\theta$$

$$= (n-1) \int_0^{\pi} \sin^{n-2}\theta - \sin^n\theta \, d\theta$$

$$I_n = (n-1) \; I_{n-2} - (n-1) \; I_n$$

$$\Rightarrow \mathbf{I_n = n\text{-}1/n \; I_{n-2}}$$

Now,

if "n" is even,

$I_n$ = n-1/n $I_{n-2}$ = n-1/n * n-3/n-2 $I_{n-4}$

$I_n$ = n-1/n * n-3/n-2 * n-5/n-4.............3/4 *1/2 * $I_0$

Since "n" is even $I_0$ = 1

= n-1/n * n-3/n-2 * n-5/n-4 ..............3/4 * 1/2 * $\int$ 1 $d\theta$

= n-1/n * n-3/n-2 * n-5/n-4 ...............3/4 * 1/2 * $\pi$

if "n" is odd ,

$I_n$ = n-1/n * n-3/n-2 * $I_{n-4}$

= n-1/n * n-3/n-2 * n-5/n-4 $I_{n-6}$

= n-1/n * n-3/n-2 * n-5/n-4..............4/5 * 2/3 * $I_1$

since "n" is odd $I_1$ = $\sin\theta$

$= n-1/n * n-3/n-2 * n-5/n-4 \ldots\ldots 4/5 * 2/3 * \int \sin\theta \, d\theta$

$= n-1/n * n-3/n-2 * n-5/n-4 \ldots\ldots 4/5 * 2/3 * 2$

consider $I_n * I_{n-1}$ ,

we know that if "n" is even then "n-1" is odd.

$I_n * I_{n-1} = n-1/n * n-3/n-2 * n-5/n-4 * \ldots\ldots 4/5 * 2/3 * 2 *$

$n-2/n-1 * n-4/n-3 * n-6/n-5 * \ldots\ldots 3/4 * 1/2 * \Pi$

$= 2\Pi / n$

we know that if "n" is odd then "n-1" is even.

$I_n * I_{n-1} = n-1/n * n-3/n-2 * n-5/n-4 \ldots\ldots 4/5 * 2/3 * 2*$

$n-2/n-1 * n-4/n-3 * n-6/n-5 \ldots\ldots 3/4 * 1/2 * \Pi$

$= 2\Pi / n$

so, whether "n" is even or odd

$$I_n * I_{n-1} = 2\Pi / n .$$

Hence,

$V_{n-1} = V_{n-1}(1) \; I_n = ( V_{n-2}(1) I_{n-1} ) \; I_n = V_{n-2} (1) \; I_n * I_{n-1}$

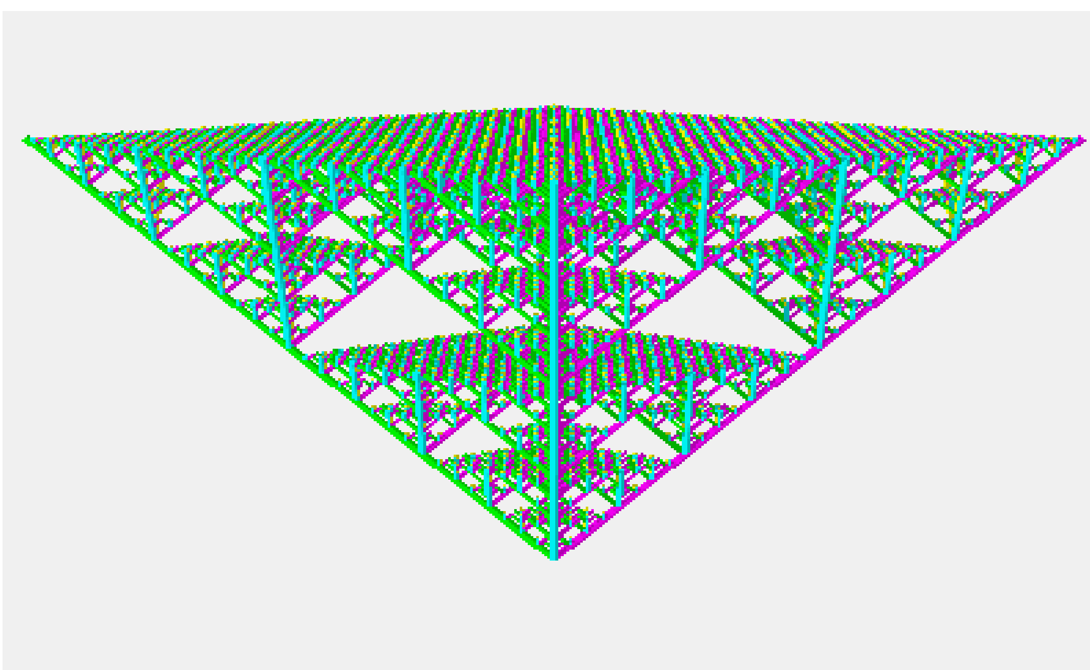
$V_n(1) = 2\Pi / n \; V_{n-2}(1)$

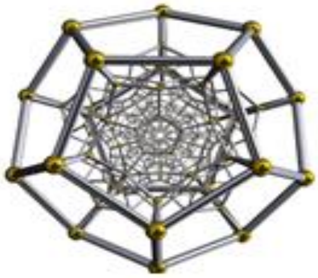
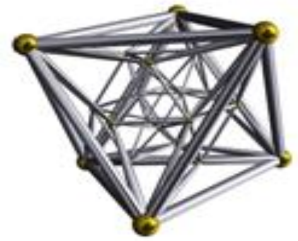
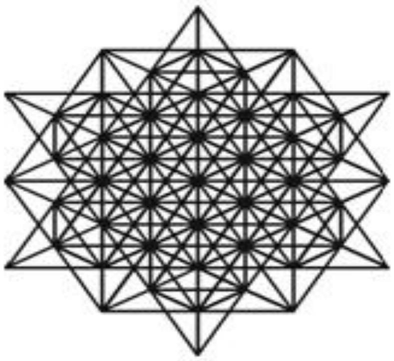
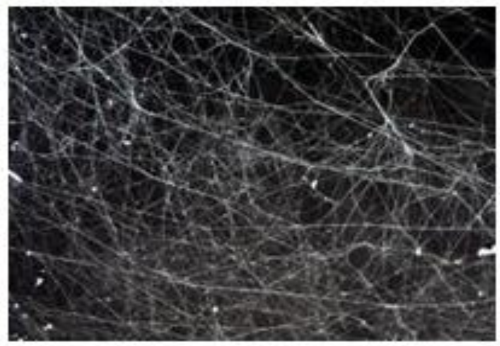
by using reduction formula for $V_n(1)$ ,
depending on whether "n" is even or odd.

$2\Pi / n * 2\Pi / n-2 * 2\Pi /n-4 * \ldots\ldots 2\Pi/2 * 1$ , for "n"even

$V_n(1) =$ $2\Pi/ n * 2\Pi/n-2 * 2\Pi/n-4 * \ldots\ldots 2\Pi/3 * 2$ , for "n"odd.

| n | V n (R) | V n (1) |
|---|---|---|
| 2 | $\prod R^2$ | $\prod = 3.14159$……. |
| 3 | $4/3\prod \ R^3$ | $4/3\prod=4.18879$……… |
| 4 | $1/2\prod^{\ 2} R^4$ | $1/2^2 = 4.9348$……….. |
| 5 | $8/15 \prod^2 R^5$ | $8/15\prod^2=5.2637$…….. |
| 6 | $1/6 \ \prod^3 R^6$ | $1/6\prod^3=5.16771$…… |
| 7 | $16/105\prod^{\ 3}R^7$ | $16/105\prod^3=4.7247$….. |
| 8 | $1/24 \ \prod^{\ 4} R^8$ | $1/24\prod^4=4.05871$…….. |



Human brain as “n” dimensional structure